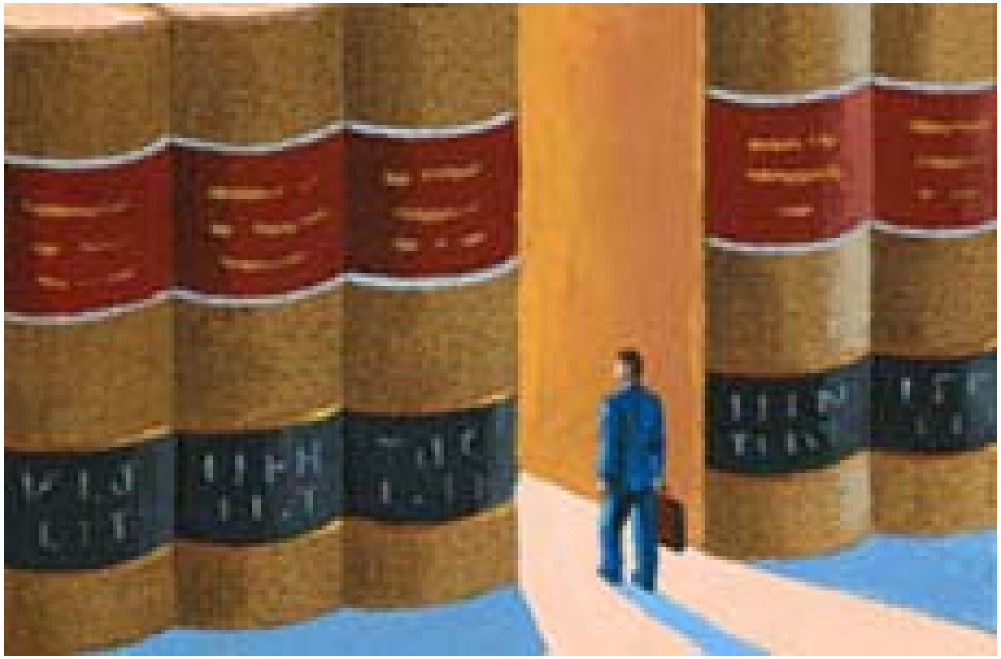
F. Description Logics – Part 2



This section is based on material from:

- Carsten Lutz, Uli Sattler: http://www.computational-logic.org/content/events/iccl-ss-2005/lectures/lutz/index.php?id=24
- Ian Horrocks: http://www.cs.man.ac.uk/~horrocks/Teaching/cs646/

Syntax für DLs (ohne concrete domains)

	Concepts	
ALC	Atomic	A, B
	Not	$\neg C$
	And	$C \sqcap D$
	Or	$C \sqcup D$
	Exists	$\exists R.C$
	For all	$\forall R.C$
Q (N)	At least	$\geq n\ R.C\ (\geq n\ R)$
	At most	$\leq n\ R.C\ (\leq n\ R)$
O	Nominal	$\{i_1,\ldots,i_n\}$

	Roles	
I	Atomic	R
	Inverse	R^-

Ontology (=Knowledge Base)

Concept Axioms (TBox)	
Subclass	$C \sqsubseteq D$
Equivalent	$C \equiv D$

	Role Axioms (RBox)	
H	Subrole	$R \sqsubseteq S$
S	Transitivity	Trans(S)

Assertional Axioms (ABox)	
Instance	C(a)
Role	R(a,b)
Same	$a = b$
Different	$a \neq b$

S = ALC + Transitivity

OWL DL = SHOIN(D) (D: concrete domain)

The Description Logic $\mathcal{ALC}$: Syntax

Atomic types:	concept names $A, B, \ldots$	(unary predicates)
	role names $R, S, \ldots$	(binary predicates)
Constructors:	- $\neg C$	(negation)
	- $C \sqcap D$	(conjunction)
	- $C \sqcup D$	(disjunction)
	- $\exists R.C$	(existential restriction)
	- $\forall R.C$	(value restriction)
Abbreviations:	- $C \rightarrow D = \neg C \sqcup D$	(implication)
	- $C \leftrightarrow D = C \rightarrow D$ $\sqcap D \rightarrow C$	(bi-implication)
	- $\top = (A \sqcup \neg A)$	(top concept)
	- $\bot = A \sqcap \neg A$	(bottom concept)

Examples

- $\text{Person} \sqcap \text{Female}$
- $\text{Person} \sqcap \exists\text{attends}.\text{Course}$
- $\text{Person} \sqcap \forall\text{attends}.(\text{Course} \to \neg\text{Easy})$
- $\text{Person} \sqcap \exists\text{teaches}.(\text{Course} \sqcap \forall\text{attended-by}.(\text{Bored} \sqcup \text{Sleeping}))$

Interpretations

Semantics based on interpretations $(\Delta^\mathcal{I}, \cdot^\mathcal{I})$, where

- $\Delta^\mathcal{I}$ is a non-empty set (the domain)
- $\cdot^\mathcal{I}$ is the interpretation function mapping

  each concept name A to a subset $A^\mathcal{I}$ of $\Delta^\mathcal{I}$ and

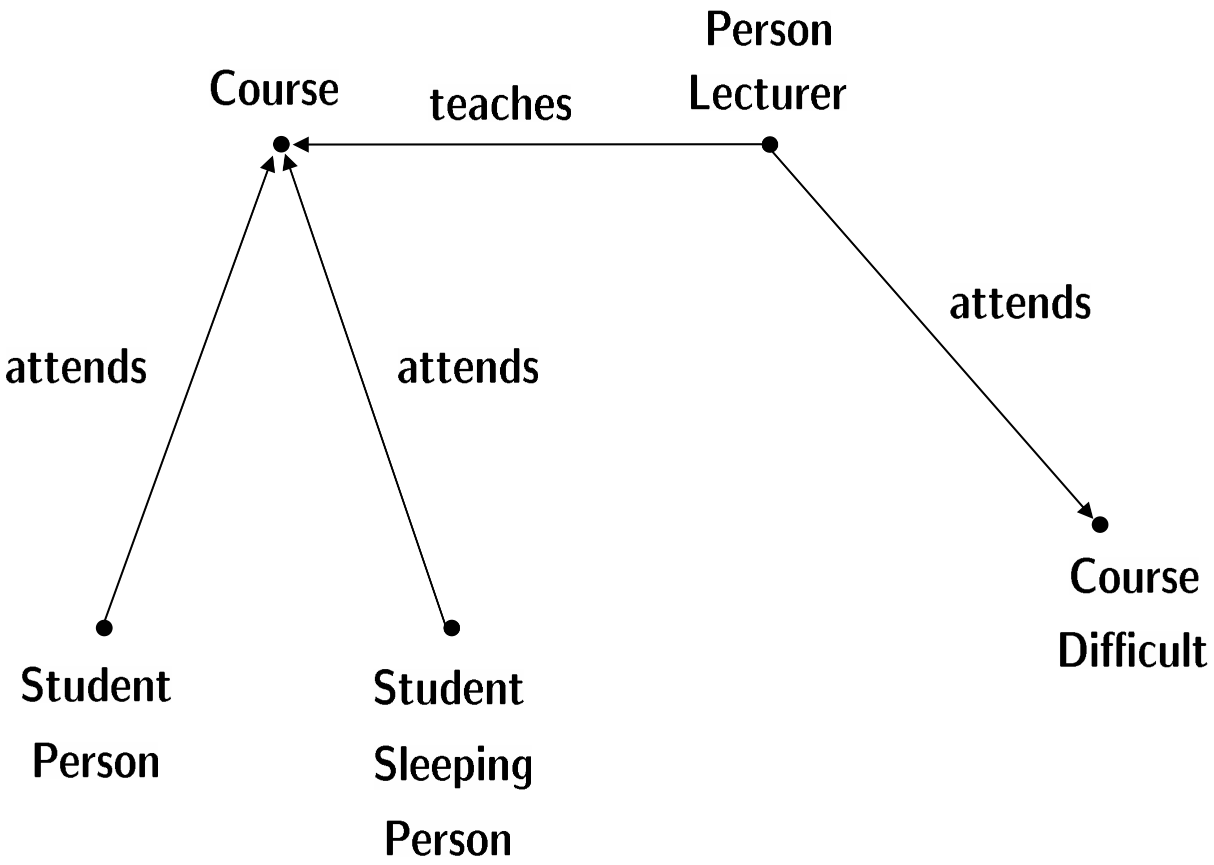
  each role name R to a binary relation $R^\mathcal{I}$ over $\Delta^\mathcal{I}$.

Intuition: interpretation is complete description of the world

Technically: interpretation is first-order structure

with only unary and binary predicates

Example

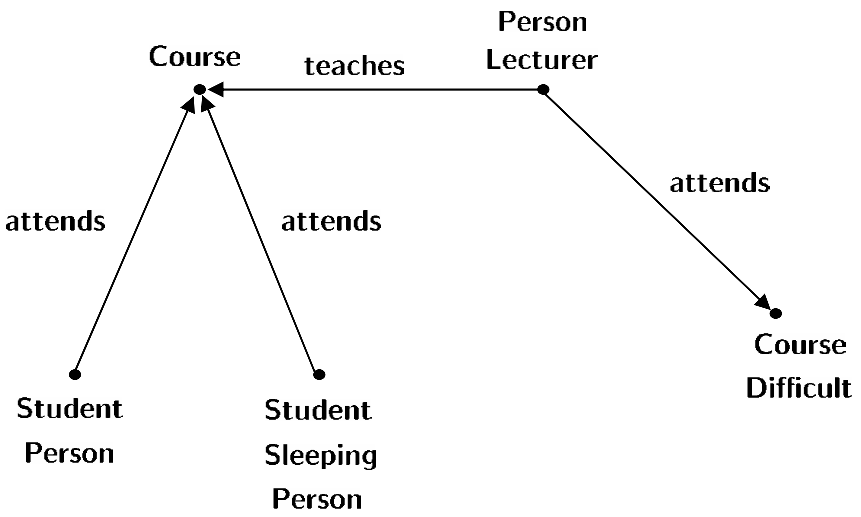

Person
Lecturer
teaches
Course
attends
attends
attends
Course
Difficult
Student
Person
Student
Sleeping
Person

Semantics of Complex Concepts

$$(\neg C)^\mathcal{I} = \Delta^\mathcal{I} \setminus C^\mathcal{I} \qquad (C \sqcap D)^\mathcal{I} = C^\mathcal{I} \cap D^\mathcal{I} \qquad (C \sqcup D)^\mathcal{I} = C^\mathcal{I} \cup D^\mathcal{I}$$

$$(\exists R.C)^\mathcal{I} = \{d \mid \text{there is an } e \in \Delta^\mathcal{I} \text{ with } (d,e) \in R^\mathcal{I} \text{ and } e \in C^\mathcal{I}\}$$

$$(\forall R.C)^\mathcal{I} = \{d \mid \text{for all } e \in \Delta^\mathcal{I}, (d,e) \in R^\mathcal{I} \text{ implies } e \in C^\mathcal{I}\}$$



Person ⊓ ∃attends.Course

Person ⊓ ∀attends.(¬Course ⊔ Difficult)

TBoxes

Capture an application's terminology means defining concepts

TBoxes are used to store concept definitions:

Syntax:

finite set of concept equations $A \doteq C$

with A concept name and C concept

left-hand sides must be unique!

Semantics:

interpretation $\mathcal{I}$ satisfies $A \doteq C$ iff $A^{\mathcal{I}} = C^{\mathcal{I}}$

$\mathcal{I}$ is model of $\mathcal{T}$ if it satisfies all definitions in $\mathcal{T}$

E.g.: Lecturer $\doteq$ Person $\sqcap$ $\exists$teaches.Course

Yields two kinds of concept names: defined and primitive

TBox: Example

TBoxes are used as ontologies:

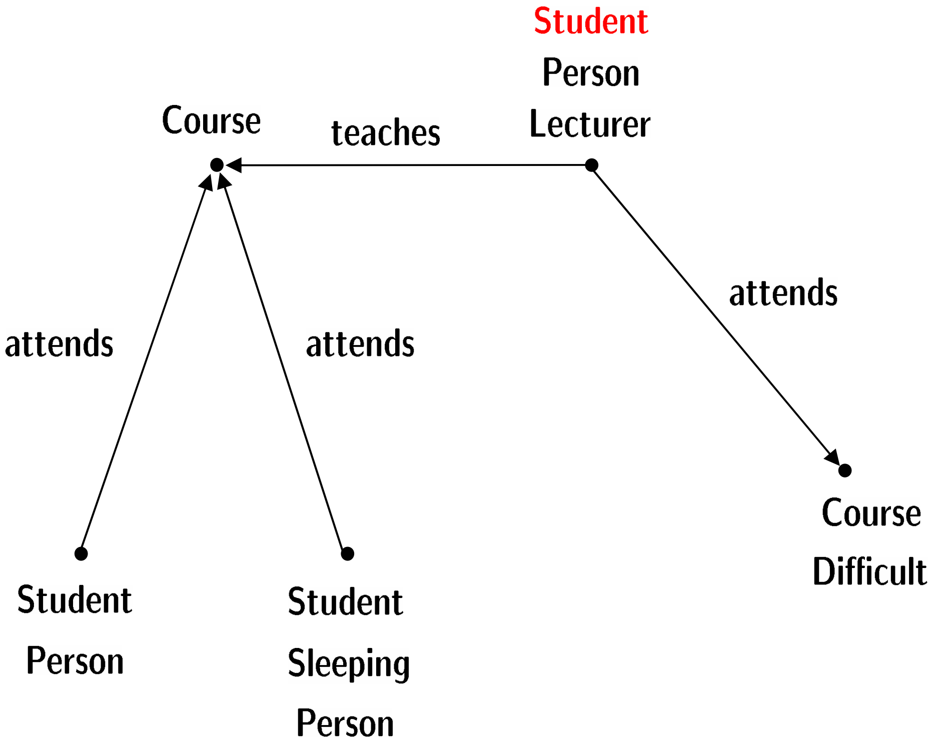
$$
\begin{aligned}
\mathsf{Woman} &\doteq \mathsf{Person} \sqcap \mathsf{Female} \\
\mathsf{Man} &\doteq \mathsf{Person} \sqcap \neg\mathsf{Woman} \\
\mathsf{Lecturer} &\doteq \mathsf{Person} \sqcap \exists\mathsf{teaches}.\mathsf{Course} \\
\mathsf{Student} &\doteq \mathsf{Person} \sqcap \exists\mathsf{attends}.\mathsf{Course} \\
\mathsf{BadLecturer} &\doteq \mathsf{Person} \sqcap \forall\mathsf{teaches}.(\mathsf{Course} \rightarrow \mathsf{Boring})
\end{aligned}
$$

TBox: Example II

A TBox restricts the set of admissible interpretations.

$$\mathsf{Lecturer} \doteq \mathsf{Person} \sqcap \exists \mathsf{teaches.Course}$$

$$\mathsf{Student} \doteq \mathsf{Person} \sqcap \exists \mathsf{attends.Course}$$



Reasoning Tasks — Subsumption

C subsumed by D w.r.t. $\mathcal{T}$ (written $C \sqsubseteq_{\mathcal{T}} D$)

iff

$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ holds for all models $\mathcal{I}$ of $\mathcal{T}$

Intuition: If $C \sqsubseteq_{\mathcal{T}} D$, then D is more general than C

Example:

$$\mathsf{Lecturer} \doteq \mathsf{Person} \sqcap \exists \mathsf{teaches}.\mathsf{Course}$$

$$\mathsf{Student} \doteq \mathsf{Person} \sqcap \exists \mathsf{attends}.\mathsf{Course}$$

Then

$$\mathsf{Lecturer} \sqcap \exists \mathsf{attends}.\mathsf{Course} \sqsubseteq_{\mathcal{T}} \mathsf{Student}$$

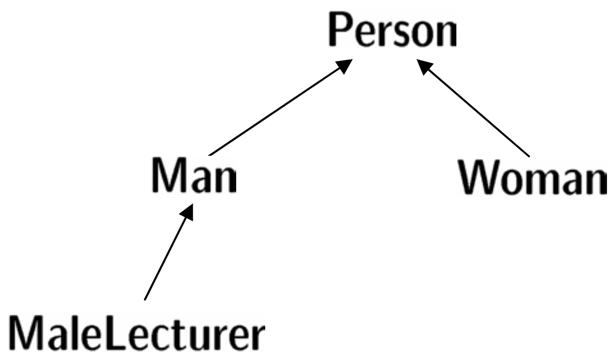
Reasoning Tasks — Classification

Classification: arrange all defined concepts from a TBox in a hierarchy w.r.t. generality

$$\mathsf{Woman} \doteq \mathsf{Person} \sqcap \mathsf{Female}$$

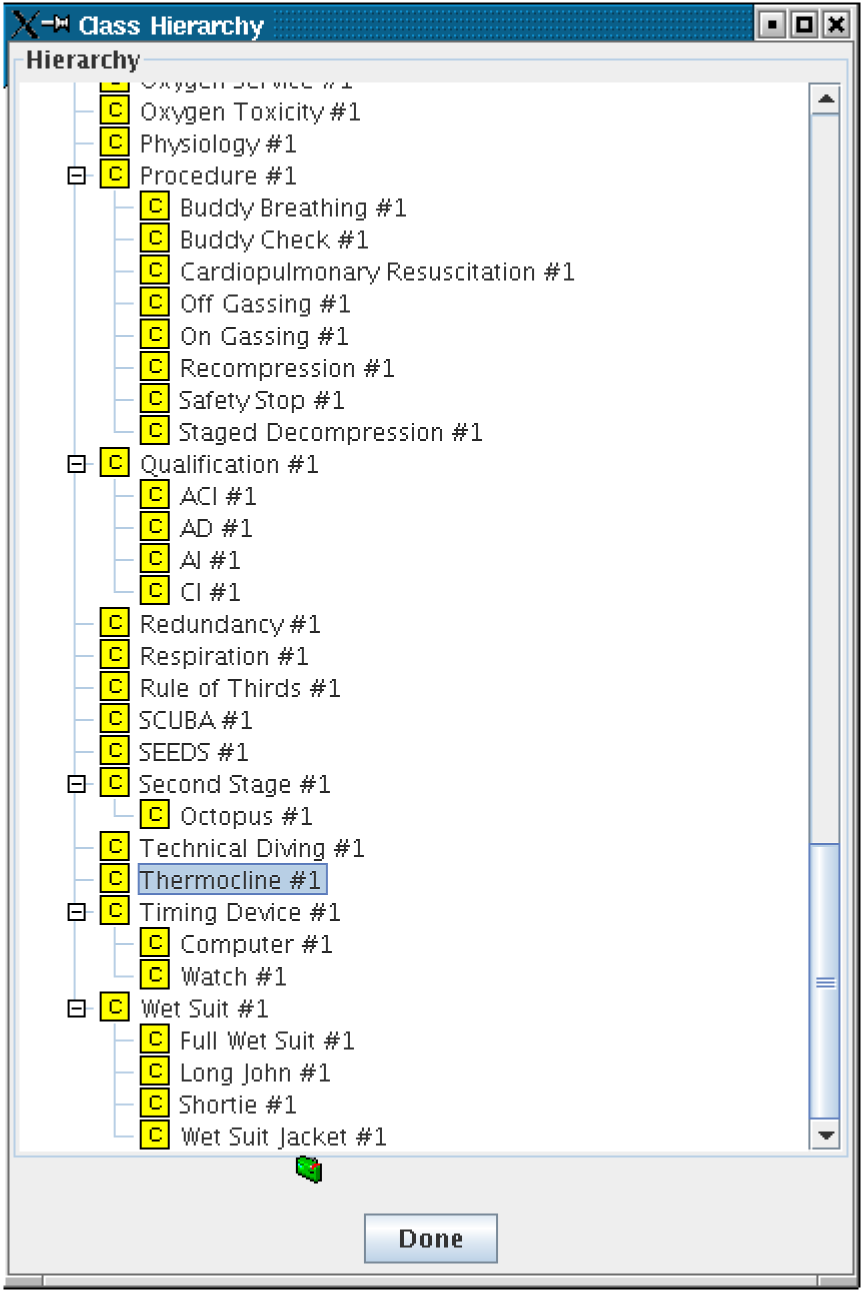
$$\mathsf{Man} \doteq \mathsf{Person} \sqcap \neg\mathsf{Woman}$$

$$\mathsf{MaleLecturer} \doteq \mathsf{Man} \sqcap \exists\mathsf{teaches}.\mathsf{Course}$$



Can be computed using multiple subsumption tests

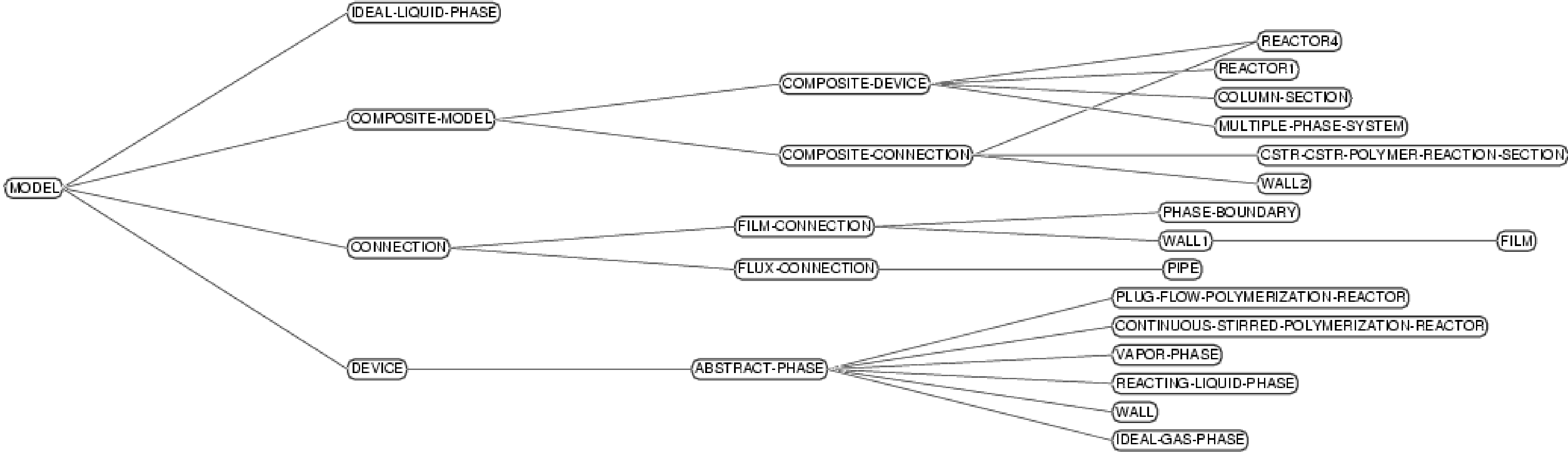
Provides a principled view on ontology for browsing, maintaining, etc.



- Oxygen Service #1
- Oxygen Toxicity #1
- Physiology #1
- Procedure #1
  - Buddy Breathing #1
  - Buddy Check #1
  - Cardiopulmonary Resuscitation #1
  - Off Gassing #1
  - On Gassing #1
  - Recompression #1
  - Safety Stop #1
  - Staged Decompression #1
- Qualification #1
  - ACI #1
  - AD #1
  - AI #1
  - CI #1
- Redundancy #1
- Respiration #1
- Rule of Thirds #1
- SCUBA #1
- SEEDS #1
- Second Stage #1
  - Octopus #1
- Technical Diving #1
- Thermocline #1
- Timing Device #1
  - Computer #1
  - Watch #1
- Wet Suit #1
  - Full Wet Suit #1
  - Long John #1
  - Shortie #1
  - Wet Suit Jacket #1

A Concept Hierarchy

Excerpt from a process engineering ontology



MODEL

IDEAL-LIQUID-PHASE

COMPOSITE-MODEL

COMPOSITE-DEVICE

REACTOR4

REACTOR1

COLUMN-SECTION

MULTIPLE-PHASE-SYSTEM

COMPOSITE-CONNECTION

CSTR-CSTR-POLYMER-REACTION-SECTION

WALL2

CONNECTION

FILM-CONNECTION

PHASE-BOUNDARY

WALL1

FILM

FLUX-CONNECTION

PIPE

DEVICE

ABSTRACT-PHASE

PLUG-FLOW-POLYMERIZATION-REACTOR

CONTINUOUS-STIRRED-POLYMERIZATION-REACTOR

VAPOR-PHASE

REACTING-LIQUID-PHASE

WALL

IDEAL-GAS-PHASE

Reasoning Tasks — Satisfiability

C is **satisfiable** w.r.t. $\mathcal{T}$ iff $\mathcal{T}$ has a model with $C^{\mathcal{I}} \neq \emptyset$

Intuition: If unsatisfiable, the concept contains a contradiction.

Example:

$$\mathsf{Woman} \doteq \mathsf{Person} \sqcap \mathsf{Female}$$

$$\mathsf{Man} \doteq \mathsf{Person} \sqcap \neg\mathsf{Woman}$$

Then $\exists\mathsf{sibling}.\mathsf{Man} \sqcap \forall\mathsf{sibling}.\mathsf{Woman}$ is unsatisfiable w.r.t. $\mathcal{T}$

Subsumption can be reduced to (un)satisfiability and vice versa:

- $C \sqsubseteq_{\mathcal{T}} D$ iff $C \sqcap \neg D$ is not satisfiable w.r.t. $\mathcal{T}$
- C is satisfiable w.r.t. $\mathcal{T}$ if not $C \sqsubseteq_{\mathcal{T}} \bot$.

Many reasoners decide satisfiability rather than subsumption.

Definitorial TBoxes

A primitive interpretation for TBox $\mathcal{T}$ interpretes

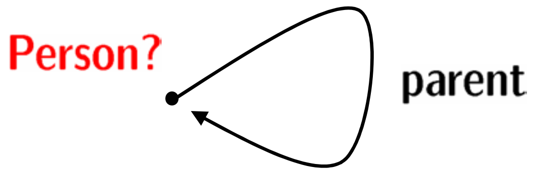
- the primitive concept names in $\mathcal{T}$
- all role names

A TBox is called definitorial if every primitive interpretation for $\mathcal{T}$ can be uniquely extended to a model of $\mathcal{T}$.

i.e.: primitive concepts (and roles) uniquely determine defined concepts

Not all TBoxes are definitorial:

$$\mathsf{Person} \doteq \exists \mathsf{parent}.\mathsf{Person}$$



Non-definitorial TBoxes describe constraints, e.g. from background knowledge

Acyclic TBoxes

TBox $\mathcal{T}$ is acyclic if there are no definitorial cycles:

~~$$\text{Lecturer} \doteq \text{Person} \sqcap \exists \text{teaches}.\text{Course}$$~~

~~$$\text{Course} \doteq \exists \text{has-title}.\text{Title} \sqcap \exists \text{tought-by}.\text{Lecturer}$$~~

Expansion of acyclic TBox $\mathcal{T}$:

exhaustively replace defined concept names with their definition

(terminates due to acyclicity)

Acyclic TBoxes are always definitorial:

first expand, then set $\quad A^{\mathcal{I}} := C^{\mathcal{I}}$ for all $A \doteq C \in \mathcal{T}$

Acyclic TBoxes II

For reasoning, acyclic TBox can be eliminated:

- to decide $C \sqsubseteq_{\mathcal{T}} D$ with $\mathcal{T}$ acyclic,
  - expand $\mathcal{T}$
  - replace defined concept names in C, D with their definition
  - decide $C \sqsubseteq D$
- analogously for satisfiability

May yield an **exponential blow-up**:

$$A_0 \doteq \forall r.A_1 \sqcap \forall s.A_1$$

$$A_1 \doteq \forall r.A_2 \sqcap \forall s.A_2$$

$$\cdots$$

$$A_{n-1} \doteq \forall r.A_n \sqcap \forall s.A_n$$

General Concept Inclusions

View of TBox as set of constraints

General TBox: finite set of general concept implications (GCIs)

$$C \sqsubseteq D$$

with both C and D allowed to be complex

e.g. Course $\sqcap$ $\forall$attended-by.Sleeping $\sqsubseteq$ Boring

Interpretation $\mathcal{I}$ is model of general TBox $\mathcal{T}$ if

$$C^{\mathcal{I}} \subseteq D^{\mathcal{I}} \text{ for all } C \sqsubseteq D \in \mathcal{T}.$$

$C \doteq D$ is abbreviation for $C \sqsubseteq D$, $D \sqsubseteq C$

e.g. Student $\sqcap$ $\exists$has-favourite.SoccerTeam $\doteq$ Student $\sqcap$ $\exists$has-favourite.Beer

Note: $C \sqsubseteq D$ equivalent to $\top \doteq C \rightarrow D$

ABoxes

ABoxes describe a snapshot of the world

An ABox is a finite set of assertions

$$a : C \qquad (a \text{ individual name, } C \text{ concept})$$

$$(a, b) : R \qquad (a, b \text{ individual names, } R \text{ role name})$$

E.g. {peter : Student, (dl-course, uli) : tought-by}

Interpretations $\mathcal{I}$ map each individual name a to an element of $\Delta^{\mathcal{I}}$.

$\mathcal{I}$ satisfies an assertion

$$a : C \quad \text{iff} \quad a^{\mathcal{I}} \in C^{\mathcal{I}}$$

$$(a, b) : R \quad \text{iff} \quad (a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$$

$\mathcal{I}$ is a model for an ABox $\mathcal{A}$ if $\mathcal{I}$ satisfies all assertions in $\mathcal{A}$.

ABoxes II

Note:

- interpretations describe the state if the world in a complete way
- ABoxes describe the state if the world in an incomplete way

$$(\text{uli}, \text{dl-course}) : \text{tought-by} \qquad \text{uli} : \text{Female}$$

does not imply

$$\text{dl-course} : \forall \text{tought-by}.\text{Female}$$

An ABox has many models!

An ABox constraints the set of admissibile models similar to a TBox

Reasoning with ABoxes

ABox consistency

Given an ABox $\mathcal{A}$ and a TBox $\mathcal{T}$, do they have a common model?

Instance checking

Given an ABox $\mathcal{A}$, a TBox $\mathcal{T}$, an individual name a, and a concept C

does $a^\mathcal{I} \in C^\mathcal{I}$ hold in all models of $\mathcal{A}$ and $\mathcal{T}$?

(written $\mathcal{A}, \mathcal{T} \models a : C$)

The two tasks are interreducible:

- $\mathcal{A}$ consistent w.r.t. $\mathcal{T}$ iff $\mathcal{A}, \mathcal{T} \not\models a : \bot$
- $\mathcal{A}, \mathcal{T} \models a : C$ iff $\mathcal{A} \cup \{a : \neg C\}$ is not consistent

Example for ABox Reasoning

ABox

$\mathsf{dumbo} : \mathsf{Mammal}$

~~$\mathsf{g23} : \mathsf{Darkgrey}$~~

$(\mathsf{dumbo}, \mathsf{g23}) : \mathsf{color}$

$\mathsf{dumbo} : \forall \mathsf{color}.\mathsf{Lightgrey}$

$\mathsf{t14} : \mathsf{Trunk}$

$(\mathsf{dumbo}, \mathsf{t14}) : \mathsf{bodypart}$

TBox

$$\mathsf{Elephant} \doteq \mathsf{Mammal} \sqcap \exists \mathsf{bodypart}.\mathsf{Trunk} \sqcap \forall \mathsf{color}.\mathsf{Grey}$$

$$\mathsf{Grey} \doteq \mathsf{Lightgrey} \sqcup \mathsf{Darkgrey}$$

$$\bot \doteq \mathsf{Lightgrey} \sqcap \mathsf{Darkgrey}$$

1. ABox is inconsistent w.r.t. TBox.

2. dumbo is an instance of Elephant

2. Tableau algorithms for $\mathcal{ALC}$ and extensions

We see a tableau algorithm for $\mathcal{ALC}$ and extend it with

① general TBoxes and

② inverse roles

Goal: Design sound and complete desicion procedures for satisfiability (and subsumption) of DLs which are well-suited for implementation purposes

A tableau algorithm for the satisfiability of $\mathcal{ALC}$ concepts

Goal: design an algorithm which takes an $\mathcal{ALC}$ concept C_0 and

1. returns *"satisfiable"* iff C_0 is satisfiable and
2. terminates, on every input,

i.e., which **decides** satisfiability of $\mathcal{ALC}$ concepts.

Recall: such an algorithm **cannot** exist for FOL since satisfiability of FOL is undecidable.

Idea: our algorithm

- is tableau-based and
- tries to construct a **model** of C_0
- by breaking C_0 down syntactically, thus
- inferring new constraints on such a model.

Preliminaries: Negation Normal Form

To make our life easier, we transform each concept C_0 into an **equivalent** C_1 in **NNF**

Equivalent: $C_0 \sqsubseteq C_1$ and $C_1 \sqsubseteq C_0$

NNF: negation occurs only in front of concept names

How? By pushing negation inwards (de Morgan et. al):

$$\begin{aligned}
\neg(C \sqcap D) &\leadsto \neg C \sqcup \neg D \\
\neg(C \sqcup D) &\leadsto \neg C \sqcap \neg D \\
\neg\neg C &\leadsto C \\
\neg\forall R.C &\leadsto \exists R.\neg C \\
\neg\exists R.C &\leadsto \forall R.\neg C
\end{aligned}$$

From now on: concepts are in NNF and
sub(C) denotes the set of all sub-concepts of C

More intuition

Find out whether $\boldsymbol{A \sqcap \exists R.B \sqcap \forall R.\neg B}$ is satisfiable...

$\boldsymbol{A \sqcap \exists R.B \sqcap \forall R.(\neg B \sqcup \exists S.E)}$

Our tableau algorithm works on a **completion tree** which

- represents a model $\mathcal{I}$: **nodes** represent elements of $\Delta^{\mathcal{I}}$

  $\rightsquigarrow$ each node x is labelled with concepts $\mathfrak{L}(x) \subseteq \mathsf{sub}(C_0)$

  $C \in \mathfrak{L}(x)$ is read as "x should be an instance of C"

  edges represent role successorship

  $\rightsquigarrow$ each edge $\langle x, y\rangle$ is labelled with a role-name from C_0

  $R \in \mathfrak{L}(\langle x, y\rangle)$ is read as "(x, y) should be in $R^{\mathcal{I}}$"
- is initialised with a single root node x_0 with $\mathfrak{L}(x_0) = \{C_0\}$
- is expanded using **completion rules**

Completion rules for $\mathcal{ALC}$

$\sqcap$-rule: if $C_1 \sqcap C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \not\subseteq \mathcal{L}(x)$

then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C_1, C_2\}$

$\sqcup$-rule: if $C_1 \sqcup C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \cap \mathcal{L}(x) = \emptyset$

then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C\}$ for some $C \in \{C_1, C_2\}$

$\exists$-rule: if $\exists S.C \in \mathcal{L}(x)$ and x has no S-successor y with $C \in \mathcal{L}(y)$,

then create a new node y with $\mathcal{L}(\langle x, y\rangle) = \{S\}$ and $\mathcal{L}(y) = \{C\}$

$\forall$-rule: if $\forall S.C \in \mathcal{L}(x)$ and there is an S-successor y of x with $C \notin \mathcal{L}(y)$

then set $\mathcal{L}(y) = \mathcal{L}(y) \cup \{C\}$

Properties of the completion rules for $\mathcal{ALC}$

We only apply rules if their application does **"something new"**

$\sqcap$-rule: if $C_1 \sqcap C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \not\subseteq \mathcal{L}(x)$

then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C_1, C_2\}$

$\sqcup$-rule: if $C_1 \sqcup C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \cap \mathcal{L}(x) = \emptyset$

then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C\}$ for some $C \in \{C_1, C_2\}$

$\exists$-rule: if $\exists S.C \in \mathcal{L}(x)$ and x has no S-successor y with $C \in \mathcal{L}(y)$,

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$\forall$-rule: if $\forall S.C \in \mathcal{L}(x)$ and there is an S-successor y of x with $C \notin \mathcal{L}(y)$

then set $\mathcal{L}(y) = \mathcal{L}(y) \cup \{C\}$

Properties of the completion rules for $\mathcal{ALC}$

The $\sqcup$-rule is **non-deterministic**:

$\sqcap$-rule: if $C_1 \sqcap C_2 \in \mathfrak{L}(x)$ and $\{C_1, C_2\} \not\subseteq \mathfrak{L}(x)$

then set $\mathfrak{L}(x) = \mathfrak{L}(x) \cup \{C_1, C_2\}$

$\sqcup$-rule: if $C_1 \sqcup C_2 \in \mathfrak{L}(x)$ and $\{C_1, C_2\} \cap \mathfrak{L}(x) = \emptyset$

then set $\mathfrak{L}(x) = \mathfrak{L}(x) \cup \{C\}$ for some $C \in \{C_1, C_2\}$

$\exists$-rule: if $\exists S.C \in \mathfrak{L}(x)$ and x has no S-successor y with $C \in \mathfrak{L}(y)$,

then create a new node y with $\mathfrak{L}(\langle x, y\rangle) = \{S\}$ and $\mathfrak{L}(y) = \{C\}$

$\forall$-rule: if $\forall S.C \in \mathfrak{L}(x)$ and there is an S-successor y of x with $C \notin \mathfrak{L}(y)$

then set $\mathfrak{L}(y) = \mathfrak{L}(y) \cup \{C\}$

Last details on tableau algorithm for $\mathcal{ALC}$

Clash: a c-tree contains a **clash** if it has a node x with $\bot \in \mathfrak{L}(x)$ or $\{A, \neg A\} \subseteq \mathfrak{L}(x)$ — otherwise, it is **clash-free**

Complete: a c-tree is **complete** if none of the completion rules can be applied to it

Answer behaviour: when started for C_0 (in NNF!), the tableau algorithm

- is **initialised** with a single root node x_0 with $\mathfrak{L}(x_0) = \{C_0\}$
- repeatedly applies the **completion rules** (in whatever order it likes)
- **answer** "C_0 *is satisfiable*" iff the completion rules **can** be applied **in such a way that** it results in a complete and clash-free c-tree (careful: this is non-deterministic)

...go back to examples

Properties of our tableau algorithm

Lemma: Let C_0 an $\mathcal{ALC}$-concept in NNF. Then

1. the algorithm terminates when applied to C_0 and
2. the rules can be applied such that they generate a clash-free and complete completion tree iff C_0 is satisfiable.

Corollary:

1. Our tableau algorithm **decides satisfiability** and **subsumption** of $\mathcal{ALC}$.
2. Satisfiability (and subsumption) in $\mathcal{ALC}$ is decidable in **PSpace**.
3. $\mathcal{ALC}$ has the **finite model property**
   i.e., every satisfiable concept has a **finite** model.
4. $\mathcal{ALC}$ has the **tree model property**
   i.e., every satisfiable concept has a **tree** model.
5. $\mathcal{ALC}$ has the **finite tree model property**
   i.e., every satisfiable concept has a **finite tree** model.

Proof of the Lemma: Termination

(1) **Termination** is an immediate consequence of these observations:

1. the c-tree is constructed in a **monotonic way**,

   each rule either adds nodes or extends node labels, nothing is removed

2. node labels are restricted to subsets of **sub**(C_0) and $\#$ **sub**$(C_0) \leq |C_0|$,

   at each position in C_0, at most one sub-concepts starts

3. the c-tree is of **bounded breadth** $\leq |C_0|$,

   at most 1 successor for each $\exists R.C \in$ **sub**(C_0)

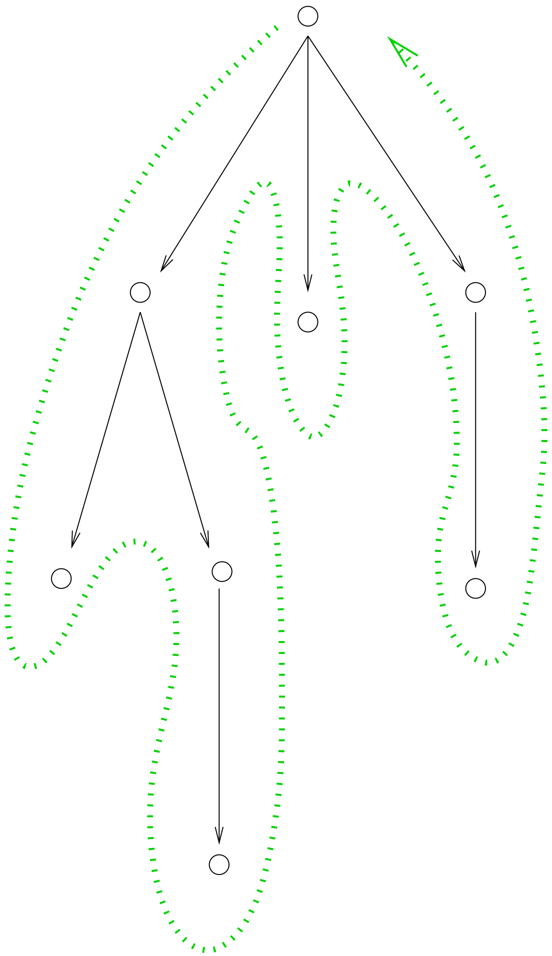
4. the c-tree is of **bounded depth** $\leq |C_0|$,

   the maximal depth of concepts in node labels decreases from a node to its successor,

   i.e., for y a successor of x: $\max\{|C| \mid C \in \mathcal{L}(y)\} < \max\{|C| \mid C \in \mathcal{L}(x)\}$

Proof of the Lemma: Termination

If we construct c-tree in depth-first manner and
re-use space for branches already visited,
mark $\exists R.C \in \mathcal{L}(x)$ with "todo" or "done"



we can run tableau algorithm in **polynomial space**:

- c-tree is of depth bounded by $|C_0|$, and
- we keep only a single branch in memory at any time.

$\leadsto$ (2) of our corollary: $\mathcal{ALC}$ is in PSpace

Proof of the Lemma: Soundness

(2) Let the algorithm stop with a complete and clash-free c-tree.
From this, define an interpretation $\mathcal{I}$ as follows:

$$\begin{aligned} \Delta^{\mathcal{I}} &:= \{x \mid x \text{ is a node in c-tree}\} \\ A^{\mathcal{I}} &:= \{x \mid A \in \mathfrak{L}(x)\} \text{ for concept names } A \\ R^{\mathcal{I}} &:= \{(x,y) \mid y \text{ is an } R\text{-successor of } x \text{ in c-tree}\} \end{aligned}$$

and show, by induction on structure of concepts, for all $x \in \Delta^{\mathcal{I}}$, $D \in \mathsf{sub}(C_0, \mathcal{T})$:

$$D \in \mathfrak{L}(x) \text{ implies } x \in D^{\mathcal{I}}$$

➜ concept names D: by definition of $\mathcal{I}$

➜ for negated concept names D: due to clash-freeness and induction

➜ for conjunctions/disjunctions/existential restrictions/universal restrictions D: due to completeness and by induction

⇝ since C_0 is in label of root node, $\mathcal{I}$ is a model of C_0

Proof of the Lemma: Completeness

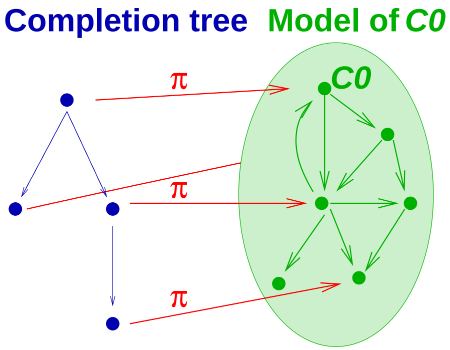
(3) Let C_0 be satisfiable, and let $\mathcal{I}$ be a model of it with $a_0 \in C_0^{\mathcal{I}}$.

Use $\mathcal{I}$ to steer the application of the (only non-deterministic) $\sqcup$-rule:

Inductively define a total mapping π :

start with $\pi(x_0) = a_0$, and show that

each rule can be applied such that $(*)$ is preserved



$$(*) \quad \text{if } C \in \mathcal{L}(x), \text{ then } \pi(x) \in C^{\mathcal{I}}$$
$$\quad \text{if } y \text{ is an } R\text{-succ. of } x, \text{ then } \langle \pi(x), \pi(y) \rangle \in R^{\mathcal{I}}$$

- easy for $\sqcap$- and $\forall$-rule,
- for $\exists$-rule, we need to extend π to the newly created R-successor
- for $\sqcup$-rule, if $C_1 \sqcup C_2 \in \mathcal{L}(x)$, $(*)$ implies that $\pi(x) \in (C_1 \sqcup C_2)^{\mathcal{I}}$
  ⇝ we can choose C_i with $\pi(x) \in C_i^{\mathcal{I}}$ to add to $\mathcal{L}(x)$ and thus preserve $(*)$

⇝ easy to see: $(*)$ implies that c-tree is **clash-free**

Proof of the Lemma: Harvest

Look again at the model $\mathcal{I}$ constructed for a clash-free, complete c-tree:

$\mathcal{I}$ is
- **finite** because c-tree has finitely many nodes
- a **tree** because c-tree is a tree

Hence we get Corollary (3) – (5) for free from our proof:

C_0 is satisfiable
$\rightsquigarrow$ tableau algorithm stops with clash-free, complete c-tree
$\rightsquigarrow$ C_0 has a finite tree model.

Extend tableau algorithm to $\mathcal{ALC}$ with general TBoxes

Recall:

- **Concept inclusion:** of the form $C \dot{\sqsubseteq} D$ for C, D (complex) concepts
- **(General) TBox:** a finite set of concept inclusions
- $\mathcal{I}$ **satisfies** $C \dot{\sqsubseteq} D$ iff $C^\mathcal{I} \subseteq D^\mathcal{I}$
- $\mathcal{I}$ **is a model of TBox** $\mathcal{T}$ iff $\mathcal{I}$ satisfies each concept equation in $\mathcal{T}$
- C_0 **is satisfiable w.r.t.** $\mathcal{T}$ iff there is a model $\mathcal{I}$ of $\mathcal{T}$ with $C_0^\mathcal{I} \neq \emptyset$

Goal – Lemma: Let C_0 an $\mathcal{ALC}$-concept and $\mathcal{T}$ be a an $\mathcal{ALC}$-TBox. Then

1. the algorithm terminates when applied to $\mathcal{T}$ and C_0 and
2. the rules can be applied such that they generate a clash-free and complete completion tree iff C_0 is satisfiable w.r.t. $\mathcal{T}$.

Extend tableau algorithm to $\mathcal{ALC}$ with general TBoxes: Preliminaries

We extend our tableau algorithm by adding a **new completion rule**:

- remember that nodes represent elements of $\Delta^{\mathcal{I}}$ and
- if $C \dot{\sqsubseteq} D \in \mathcal{T}$, then for each element x in a model $\mathcal{I}$ of $\mathcal{T}$

  if $x \in C^{\mathcal{I}}$, then $x \in D^{\mathcal{I}}$

  hence $x \in (\neg C)^{\mathcal{I}}$ or $x \in D^{\mathcal{I}}$

  $x \in (\neg C \sqcup D)^{\mathcal{I}}$

  $x \in (\textbf{NNF}(\neg C \sqcup D))^{\mathcal{I}}$

  for **NNF**(E) the negation normal form of E

Completion rules for $\mathcal{ALC}$ with TBoxes

$\sqcap$-rule: if $C_1 \sqcap C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \not\subseteq \mathcal{L}(x)$

then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C_1, C_2\}$

$\sqcup$-rule: if $C_1 \sqcup C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \cap \mathcal{L}(x) = \emptyset$

then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C\}$ for some $C \in \{C_1, C_2\}$

$\exists$-rule: if $\exists S.C \in \mathcal{L}(x)$ and x has no S-successor y with $C \in \mathcal{L}(y)$,

then create a new node y with $\mathcal{L}(\langle x, y\rangle) = \{S\}$ and $\mathcal{L}(y) = \{C\}$

$\forall$-rule: if $\forall S.C \in \mathcal{L}(x)$ and there is an S-successor y of x with $C \notin \mathcal{L}(y)$

then set $\mathcal{L}(y) = \mathcal{L}(y) \cup \{C\}$

$\mathcal{T}$-rule: if $C_1 \dot{\sqsubseteq} C_2 \in \mathcal{T}$ and NNF$(\neg C_1 \sqcup C_2) \notin \mathcal{L}(x)$

then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{\text{NNF}(\neg C_1 \sqcup C_2)\}$

A tableau algorithm for $\mathcal{ALC}$ with general TBoxes

Example: Consider satisfiability of C w.r.t. $\{C \dot{\sqsubseteq} \exists R.C\}$

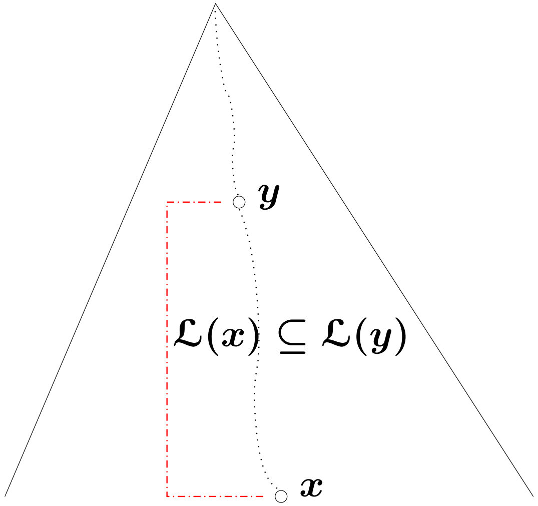
Tableau algorithm no longer terminates!

Reason: size of concepts no longer decreases along paths in a completion tree

Observation: most nodes on this path look the same and
we keep repeating ourselves

Regain termination with a "cycle-detection" technique called blocking

Intuitively, whenever we find a situation where y has to satisfy *stronger* constraints than x, we *freeze* x, i.e., block rules from being applied to x



A tableau algorithm for $\mathcal{ALC}$ with general TBoxes: Blocking

- x is **directly blocked** if it has an ancestor y with $\mathfrak{L}(x) \subseteq \mathfrak{L}(y)$
- in this case and if y is the "closest" such node to x, we say that x **is blocked by** y
- a node is **blocked** if it is directly blocked or one of its ancestors is blocked

$\oplus$ restrict the application of all rules to nodes which are not blocked

$\leadsto$ **completion rules for $\mathcal{ALC}$ w.r.t. TBoxes**

A tableau algorithm for $\mathcal{ALC}$ with general TBoxes

$\sqcap$-rule: if $C_1 \sqcap C_2 \in \mathfrak{L}(x)$, $\{C_1, C_2\} \not\subseteq \mathfrak{L}(x)$, and x is not blocked
then set $\mathfrak{L}(x) = \mathfrak{L}(x) \cup \{C_1, C_2\}$

$\sqcup$-rule: if $C_1 \sqcup C_2 \in \mathfrak{L}(x)$, $\{C_1, C_2\} \cap \mathfrak{L}(x) = \emptyset$, and x is not blocked
then set $\mathfrak{L}(x) = \mathfrak{L}(x) \cup \{C\}$ for some $C \in \{C_1, C_2\}$

$\exists$-rule: if $\exists S.C \in \mathfrak{L}(x)$, x has no S-successor y with $C \in \mathfrak{L}(y)$,
and x is not blocked
then create a new node y with $\mathfrak{L}(\langle x, y\rangle) = \{S\}$ and $\mathfrak{L}(y) = \{C\}$

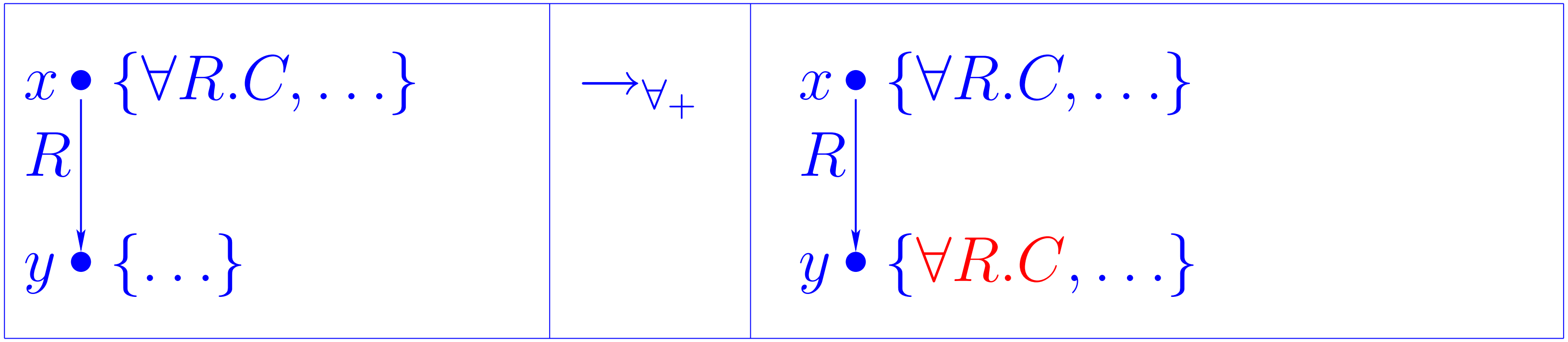
$\forall$-rule: if $\forall S.C \in \mathfrak{L}(x)$, there is an S-successor y of x with $C \notin \mathfrak{L}(y)$
and x is not blocked
then set $\mathfrak{L}(y) = \mathfrak{L}(y) \cup \{C\}$

$\mathcal{T}$-rule: if $C_1 \dot{\sqsubseteq} C_2 \in \mathcal{T}$, NNF$(\neg C_1 \sqcup C_2) \notin \mathfrak{L}(x)$
and x is not blocked
then set $\mathfrak{L}(x) = \mathfrak{L}(x) \cup \{\text{NNF}(\neg C_1 \sqcup C_2)\}$

Tableaux Rules for $\mathcal{ALC}$

$x \bullet \{C_1 \sqcap C_2, \ldots\}$	$\rightarrow_{\sqcap}$	$x \bullet \{C_1 \sqcap C_2, C_1, C_2, \ldots\}$
$x \bullet \{C_1 \sqcup C_2, \ldots\}$	$\rightarrow_{\sqcup}$	$x \bullet \{C_1 \sqcup C_2, C, \ldots\}$ for $C \in \{C_1, C_2\}$
$x \bullet \{\exists R.C, \ldots\}$	$\rightarrow_{\exists}$	$x \bullet \{\exists R.C, \ldots\}$ $R \downarrow$ $y \bullet \{C\}$
$x \bullet \{\forall R.C, \ldots\}$ $R \downarrow$ $y \bullet \{\ldots\}$	$\rightarrow_{\forall}$	$x \bullet \{\forall R.C, \ldots\}$ $R \downarrow$ $y \bullet \{C, \ldots\}$

Tableaux Rule for Transitive Roles



$x \bullet \{\forall R.C, \ldots\}$

R

$y \bullet \{\ldots\}$

$\rightarrow_{\forall_+}$

$x \bullet \{\forall R.C, \ldots\}$

R

$y \bullet \{\forall R.C, \ldots\}$

Where R is a transitive role (i.e., $(R^{\mathcal{I}})^+ = R^{\mathcal{I}}$)

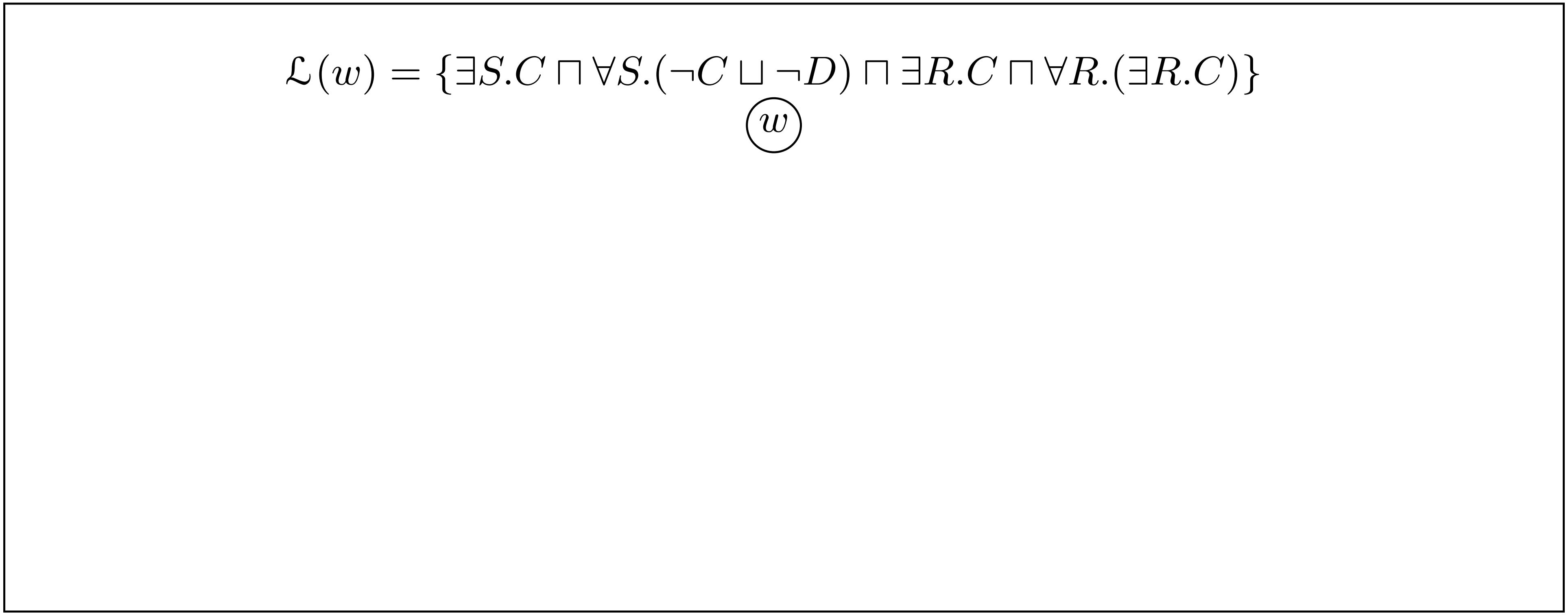
- No longer naturally terminating (e.g., if $C = \exists R.\top$)
- Need blocking
  - Simple blocking suffices for $\mathcal{ALC}$ plus transitive roles
  - I.e., do not expand node label if ancestor has superset label
  - More expressive logics (e.g., with inverse roles) need more sophisticated blocking strategies

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role

Tableaux Algorithm — Example

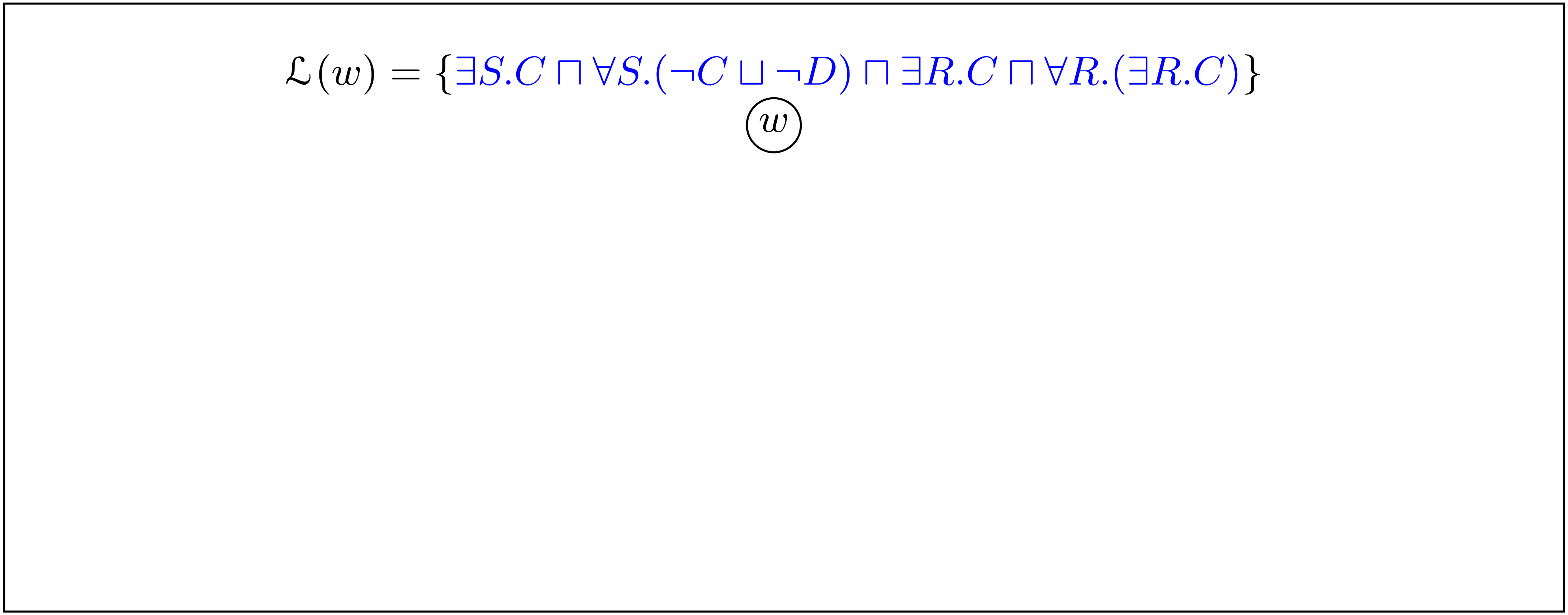
Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role



$$\mathcal{L}(w) = \{\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$$

Tableaux Algorithm — Example

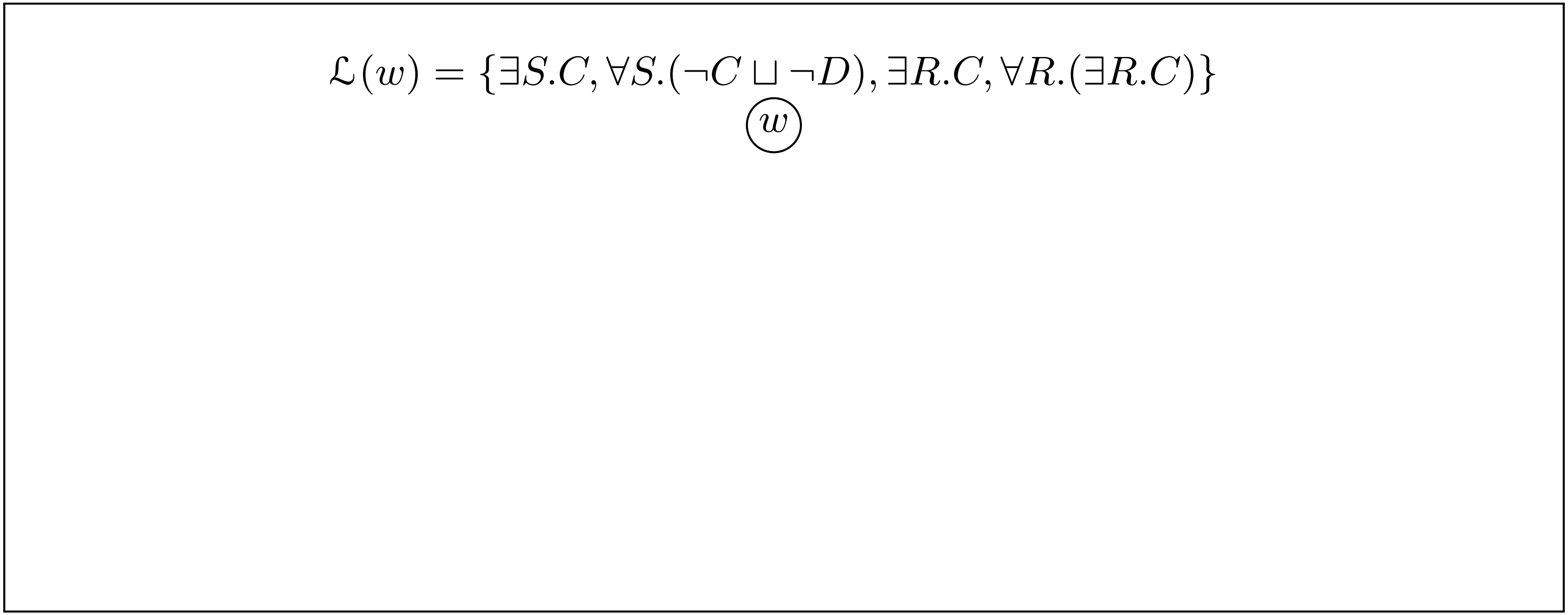
Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role



$$\mathcal{L}(w) = \{\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$$

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role



$$\mathcal{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$$

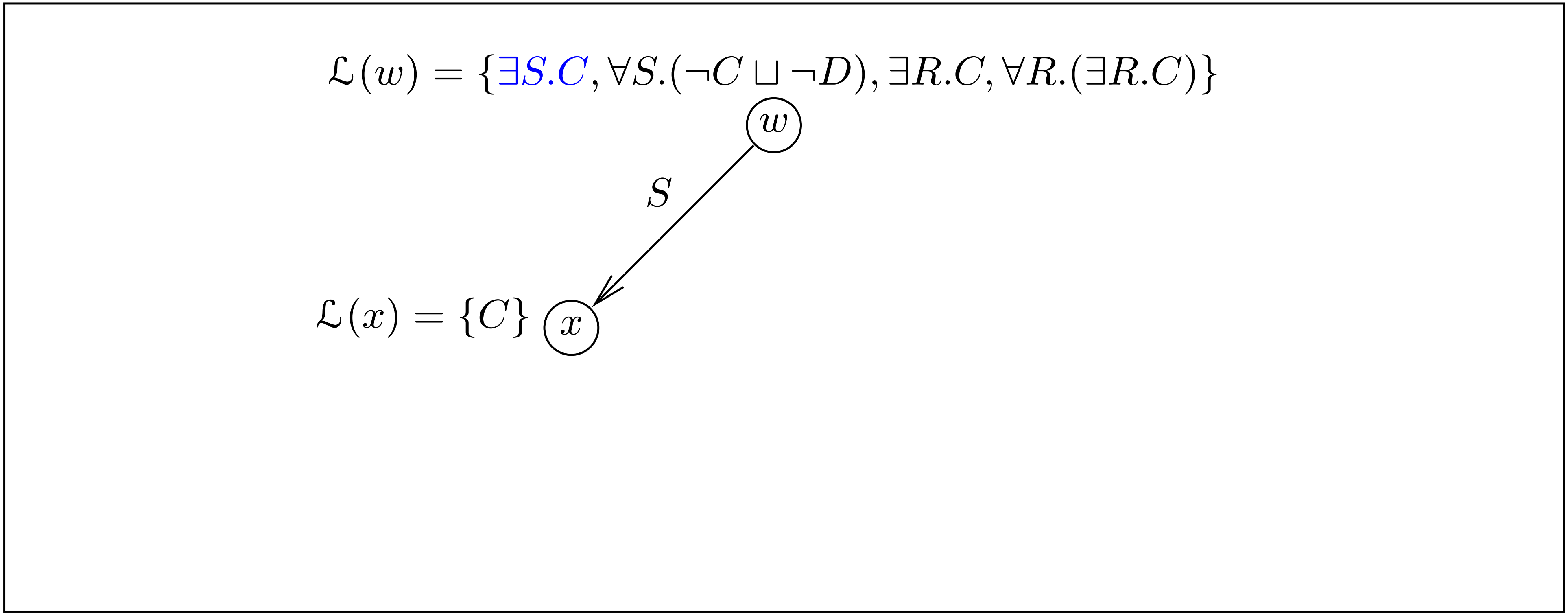
Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role

$$\mathcal{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$$

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role

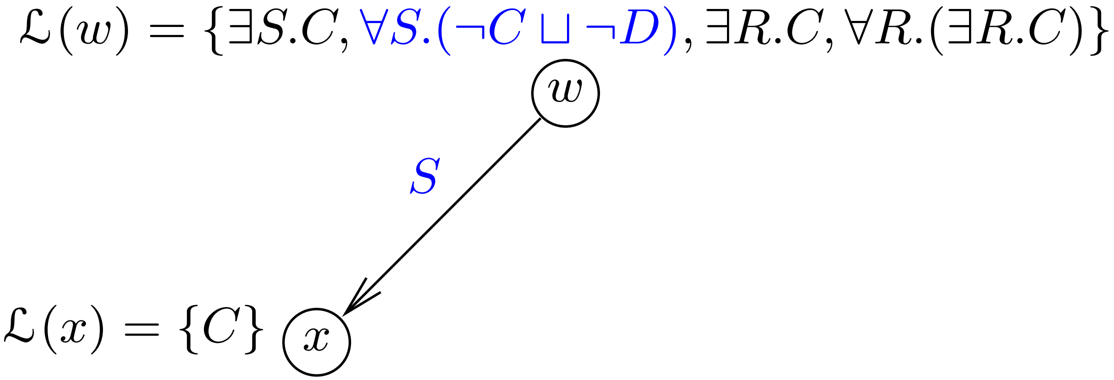


$\mathcal{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$

$\mathcal{L}(x) = \{C\}$

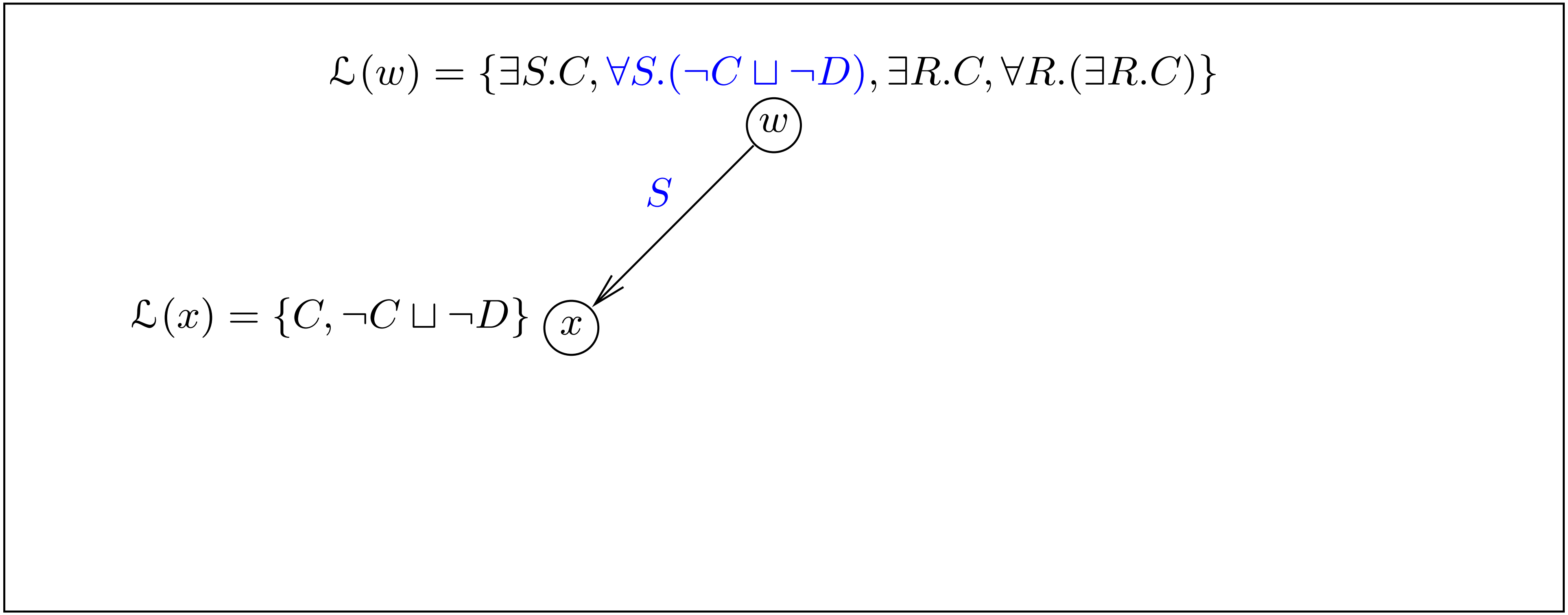
Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role


$\mathcal{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$
w
S
$\mathcal{L}(x) = \{C\}$
x

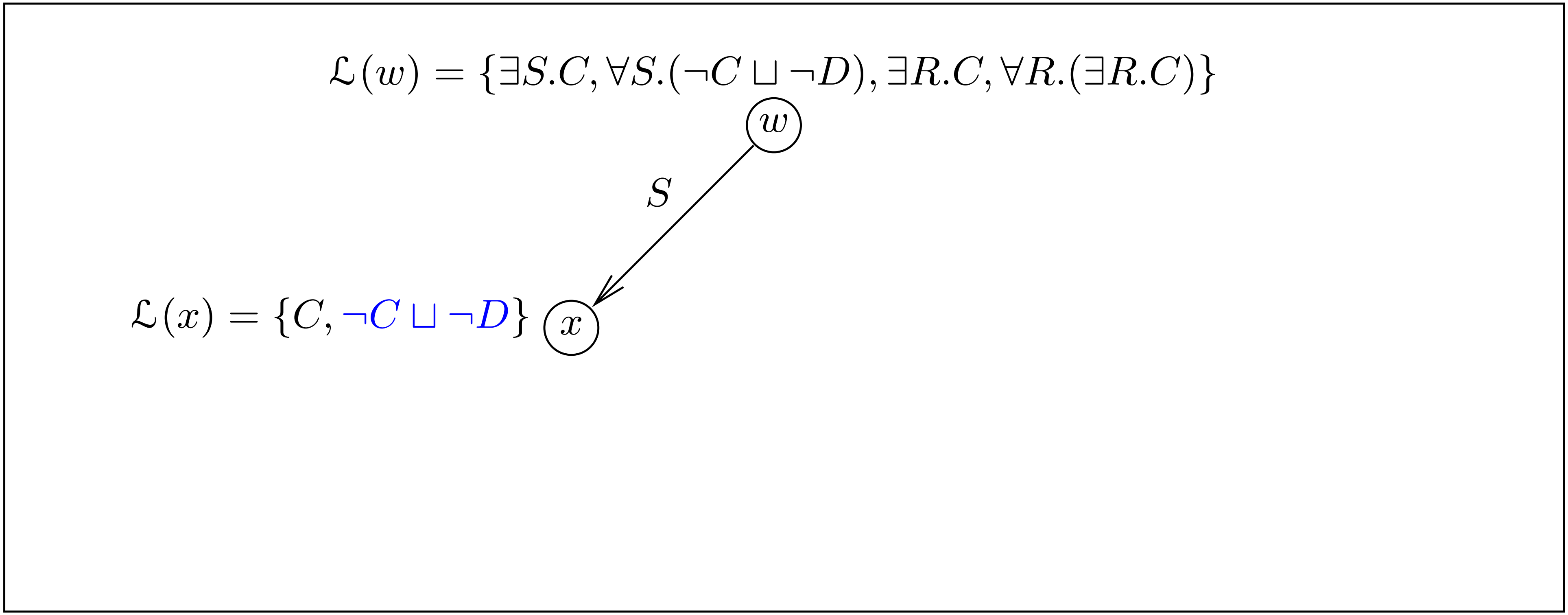
Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role


$\mathcal{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$
w
S
$\mathcal{L}(x) = \{C, \neg C \sqcup \neg D\}$
x

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role

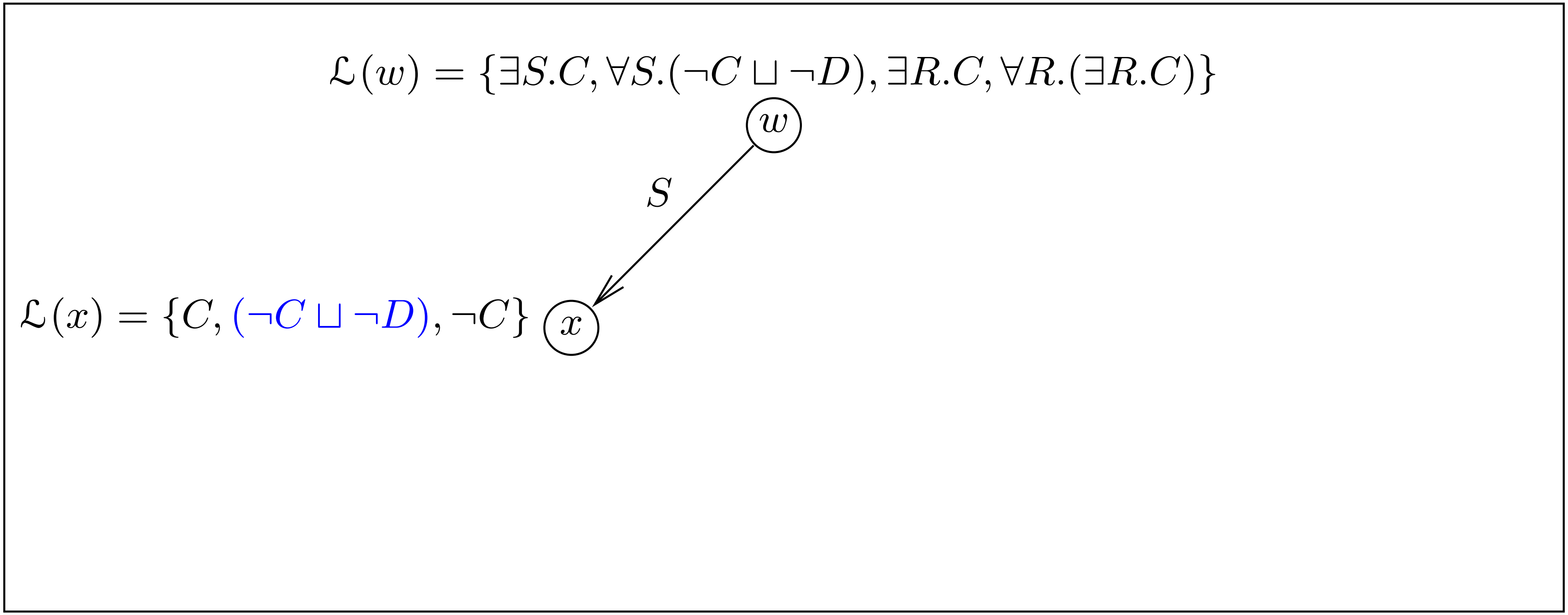


$\mathcal{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$

$\mathcal{L}(x) = \{C, \neg C \sqcup \neg D\}$

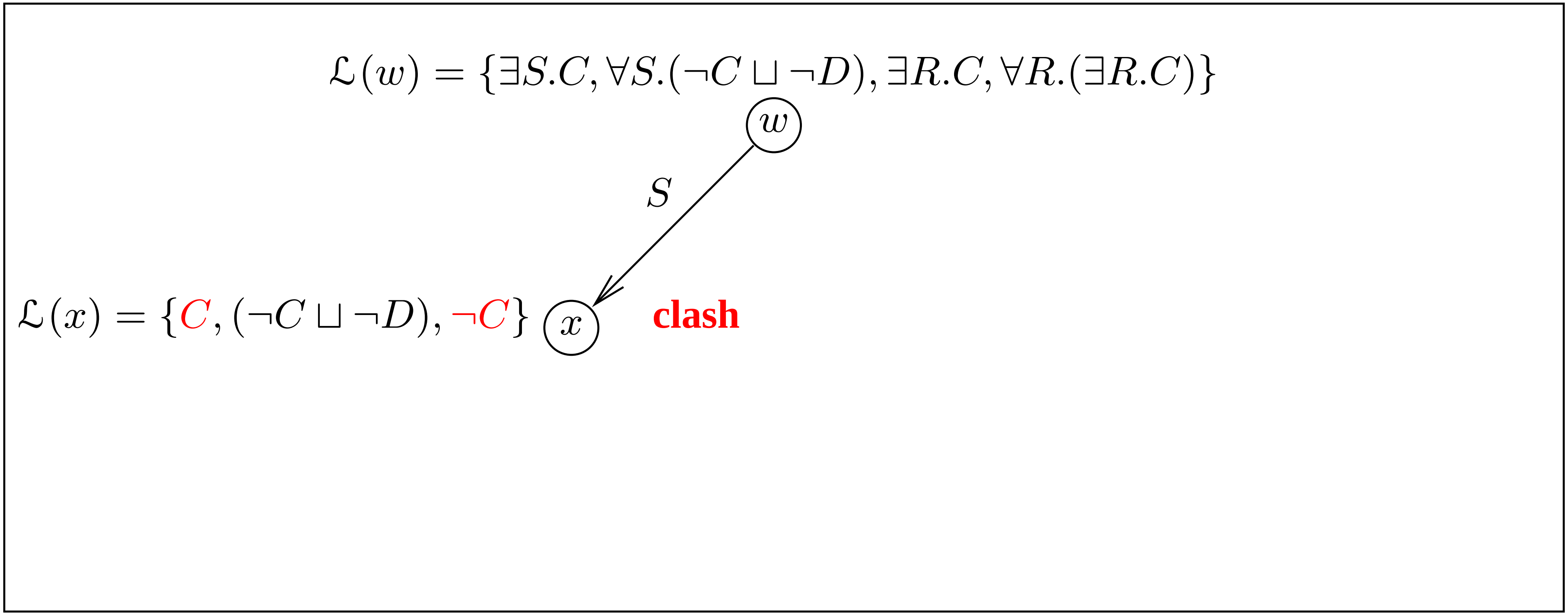
Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role


$\mathcal{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$
w
S
$\mathcal{L}(x) = \{C, (\neg C \sqcup \neg D), \neg C\}$
x

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role



$\mathcal{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$

w

S

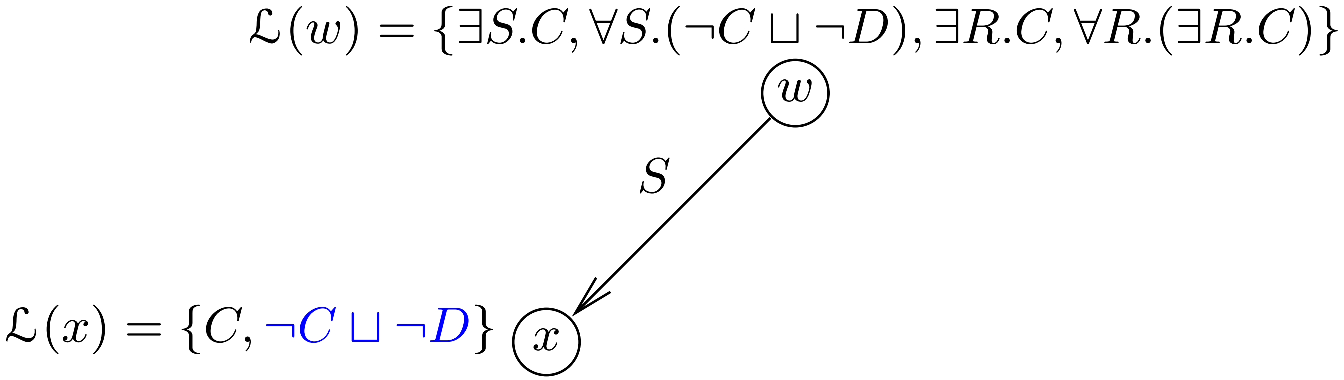
$\mathcal{L}(x) = \{C, (\neg C \sqcup \neg D), \neg C\}$

x

clash

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role

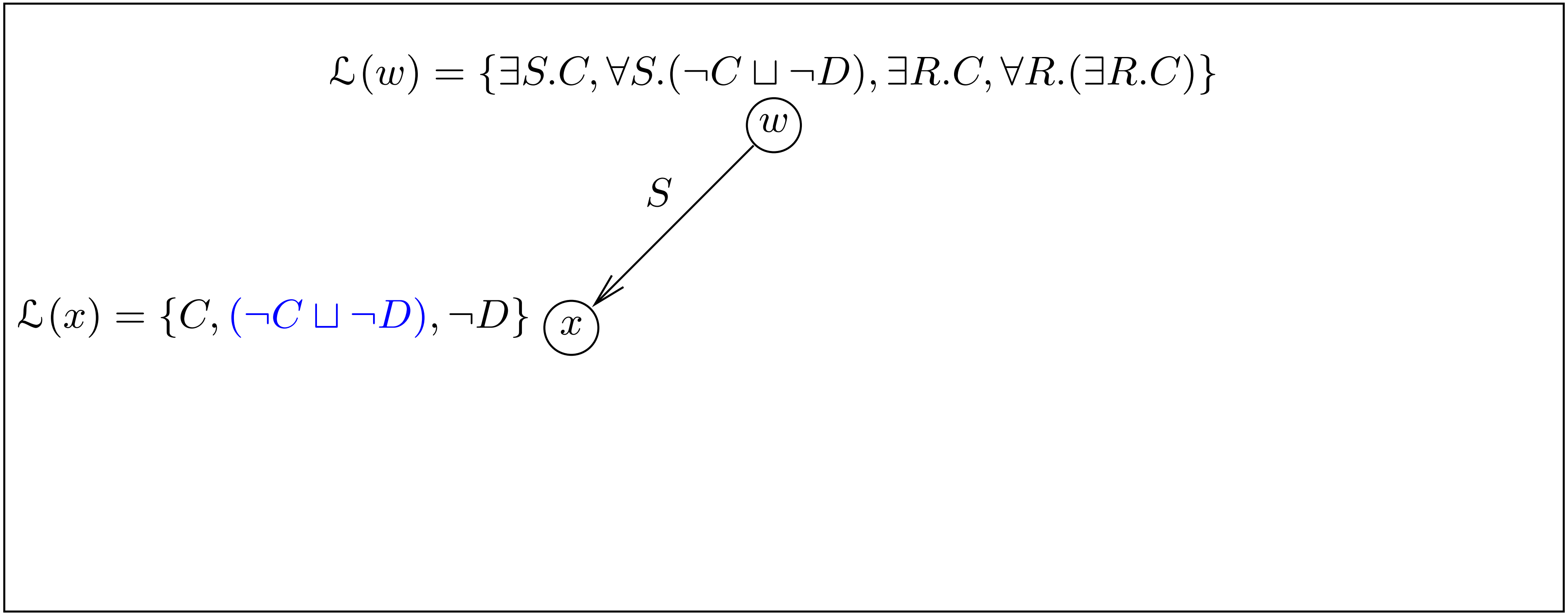


$\mathcal{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$

$\mathcal{L}(x) = \{C, \neg C \sqcup \neg D\}$

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role



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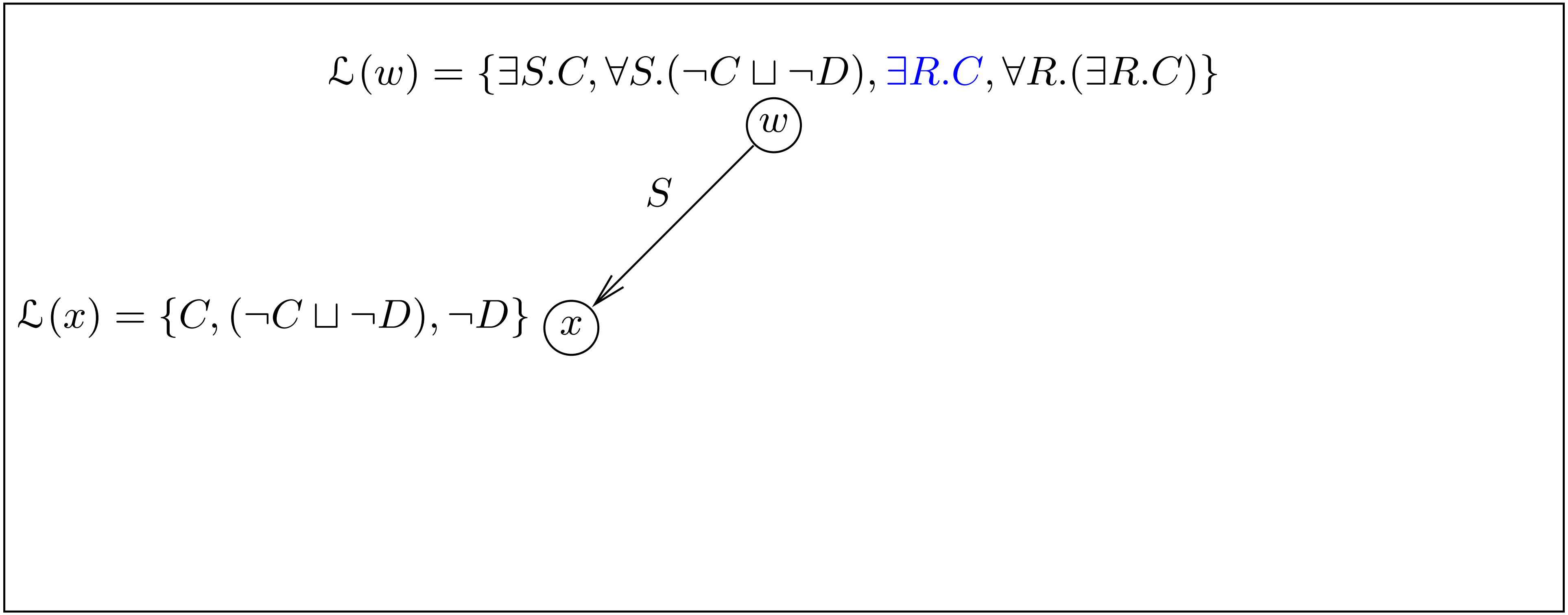
$\mathcal{L}(x) = \{C, (\neg C \sqcup \neg D), \neg D\}$

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role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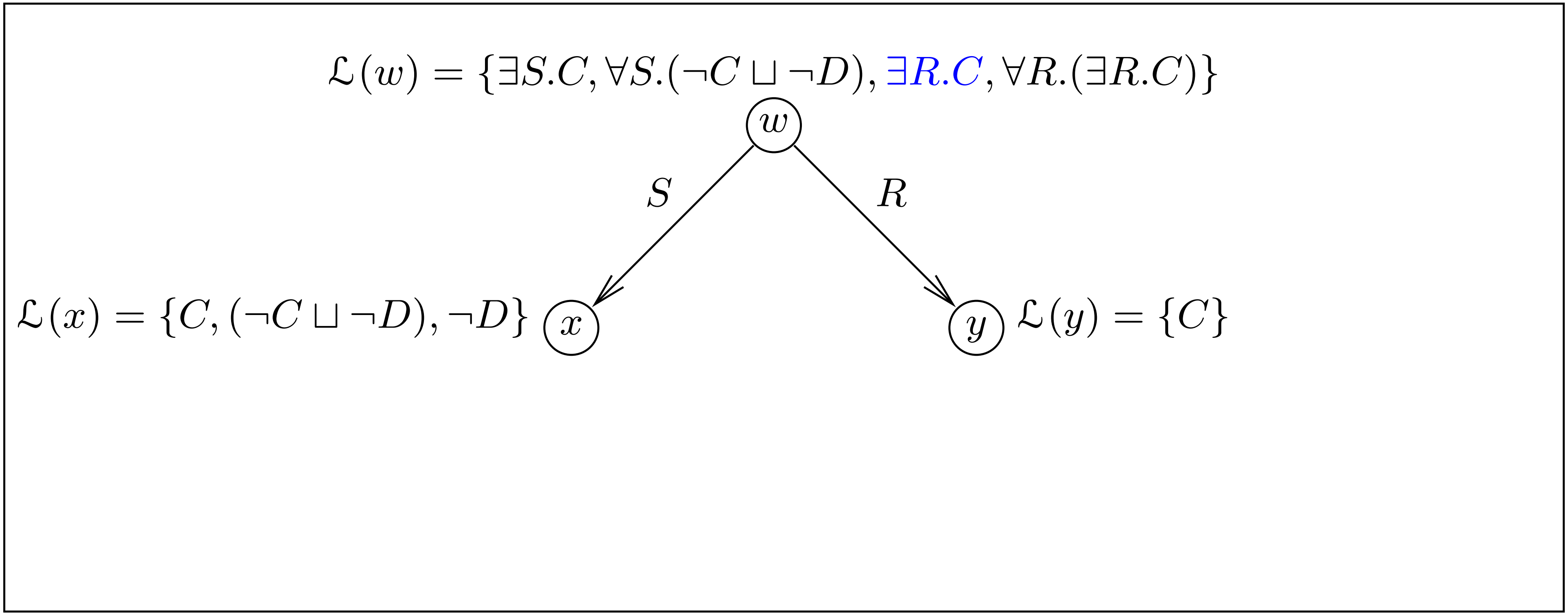
w
S
$\mathcal{L}(x) = \{C, (\neg C \sqcup \neg D), \neg D\}$
x

Tableaux Algorithm — Example

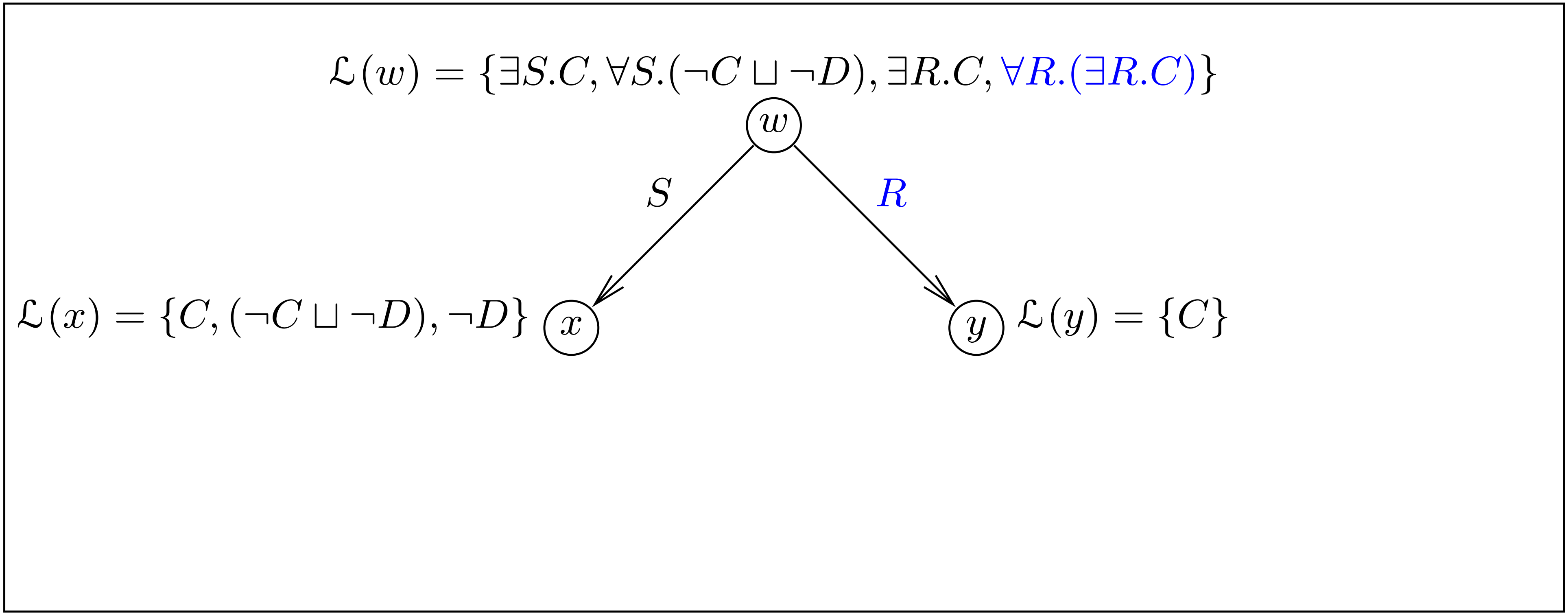
Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role



$\mathcal{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$
w
S
R
$\mathcal{L}(x) = \{C, (\neg C \sqcup \neg D), \neg D\}$
x
y
$\mathcal{L}(y) = \{C\}$

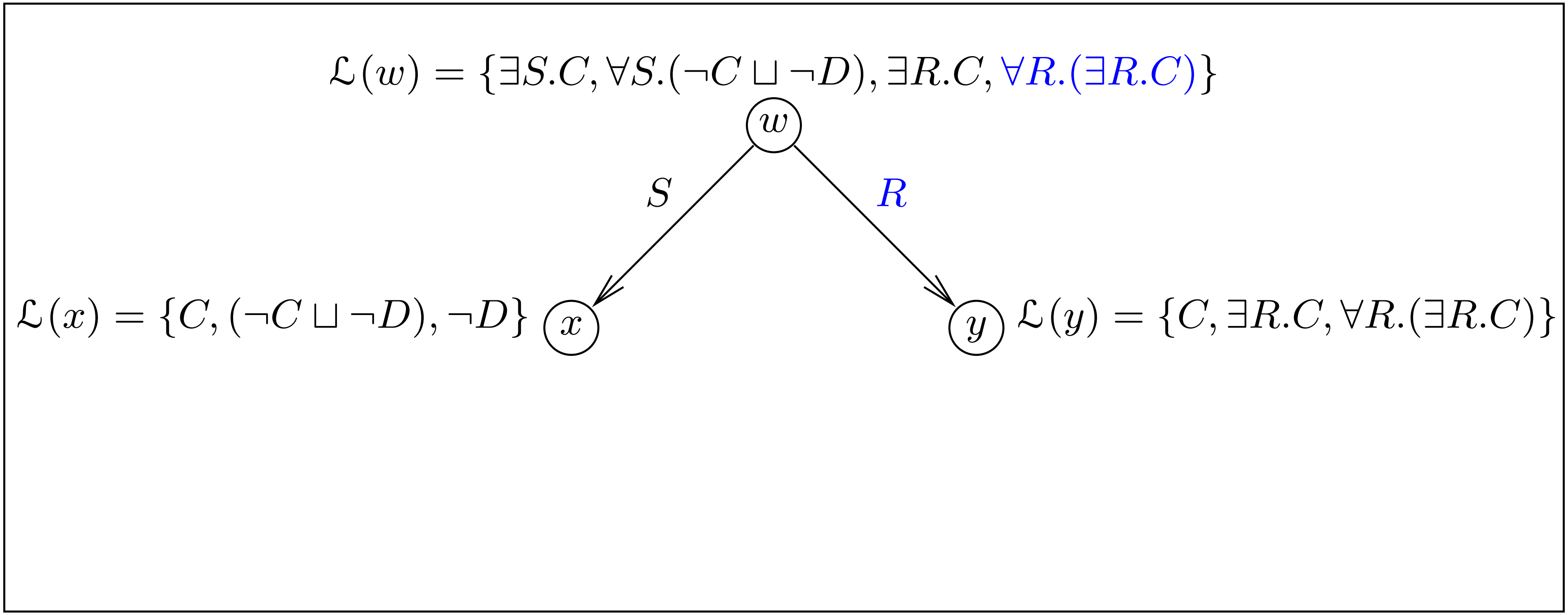
Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role


$\mathcal{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$
w
S
R
$\mathcal{L}(x) = \{C, (\neg C \sqcup \neg D), \neg D\}$
x
y
$\mathcal{L}(y) = \{C\}$

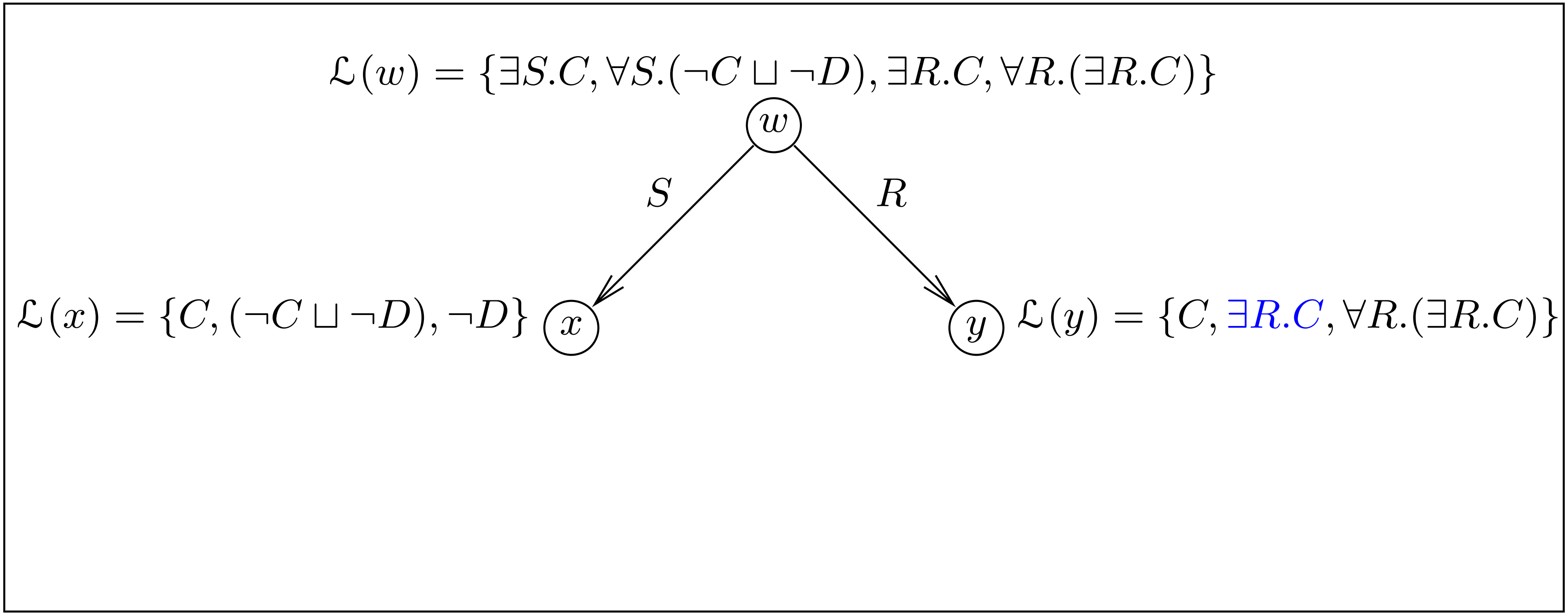
Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role


$\mathcal{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$
w
S
R
$\mathcal{L}(x) = \{C, (\neg C \sqcup \neg D), \neg D\}$
x
y
$\mathcal{L}(y) = \{C, \exists R.C, \forall R.(\exists R.C)\}$

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role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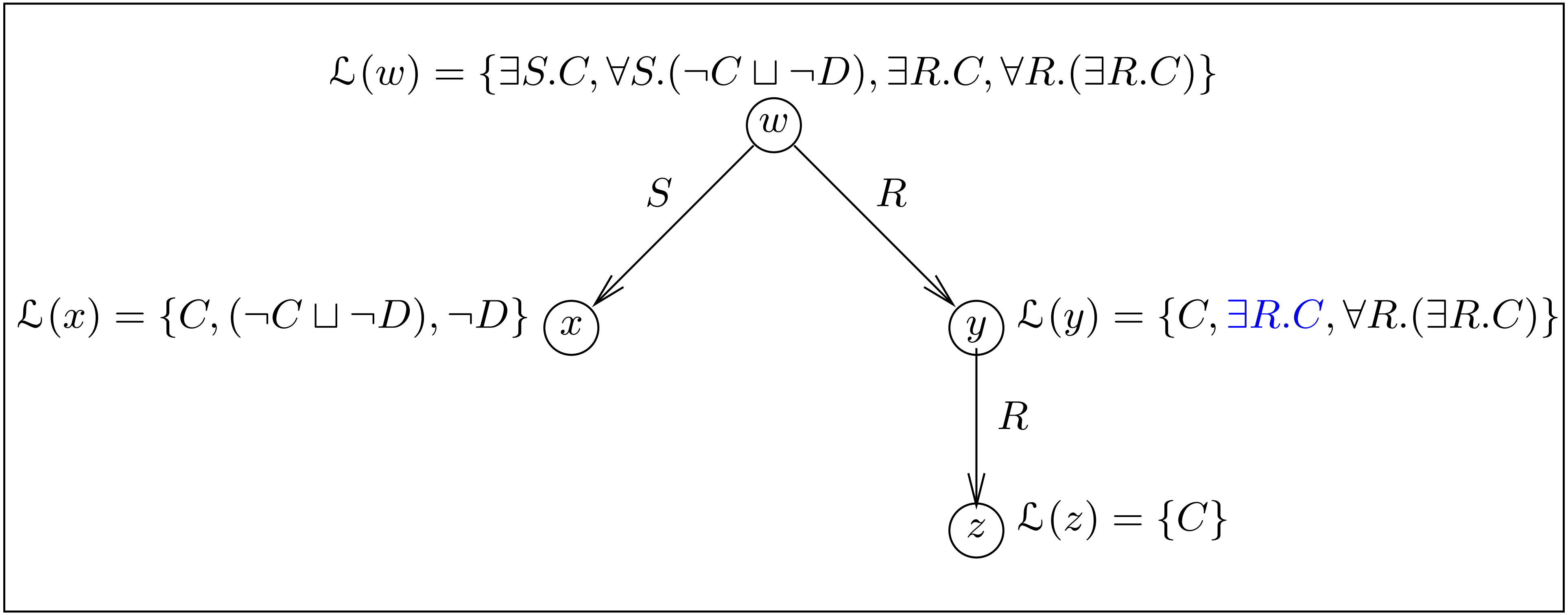
w
S
R
$\mathcal{L}(x) = \{C, (\neg C \sqcup \neg D), \neg D\}$
x
y
$\mathcal{L}(y) = \{C, \exists R.C, \forall R.(\exists R.C)\}$

Tableaux Algorithm — Example

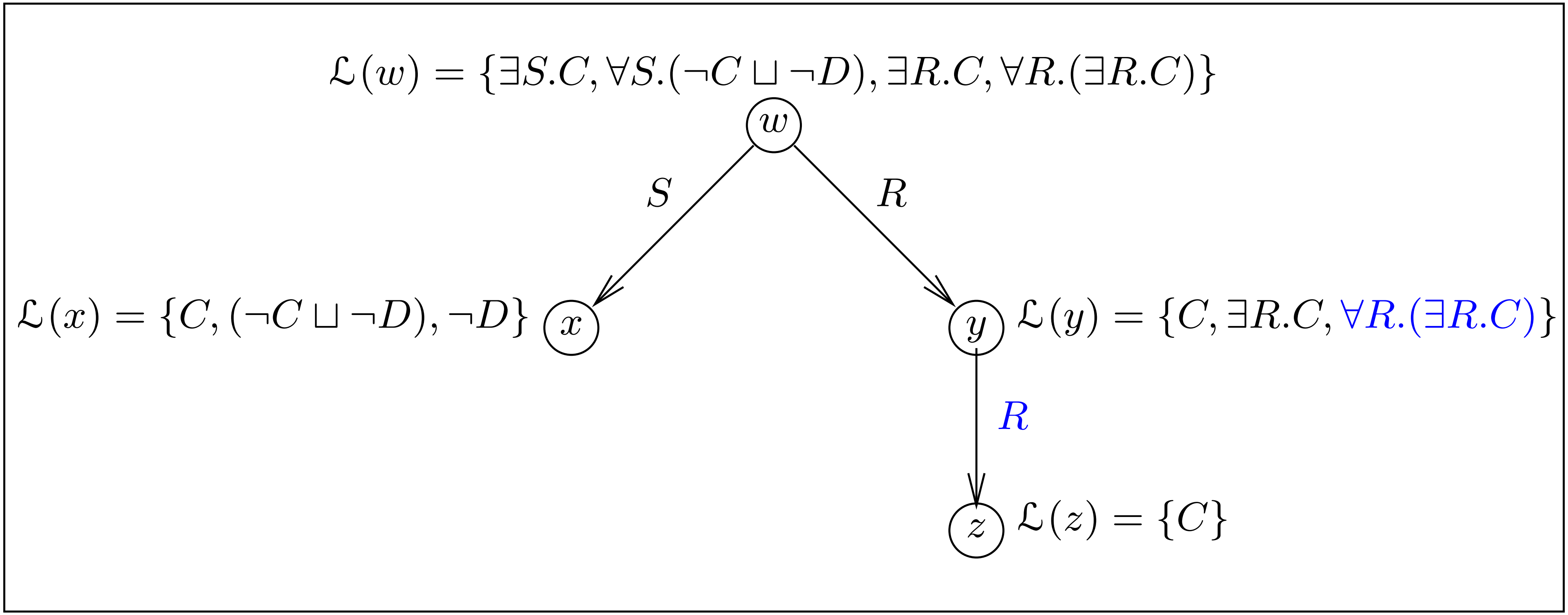
Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role



$\mathfrak{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$
w
S
R
$\mathfrak{L}(x) = \{C, (\neg C \sqcup \neg D), \neg D\}$
x
y
$\mathfrak{L}(y) = \{C, \exists R.C, \forall R.(\exists R.C)\}$
R
z
$\mathfrak{L}(z) = \{C\}$

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role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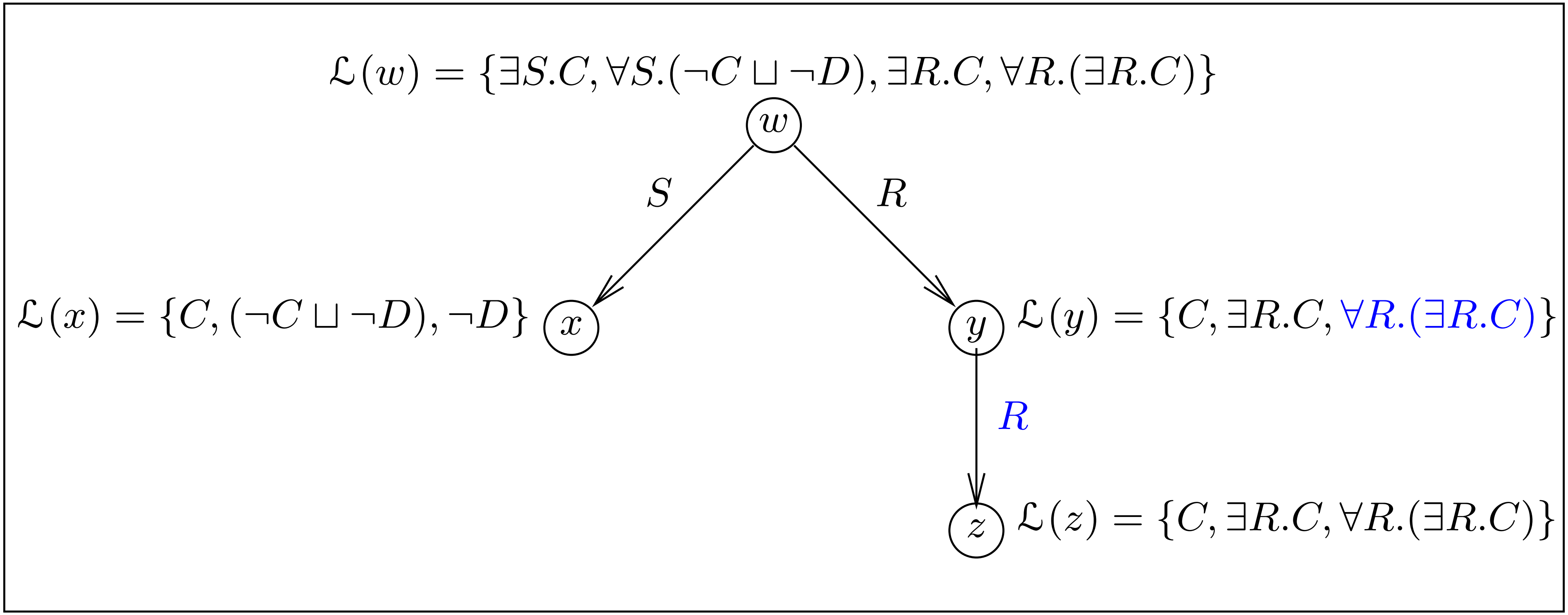
w
S
R
$\mathcal{L}(x) = \{C, (\neg C \sqcup \neg D), \neg D\}$
x
y
$\mathcal{L}(y) = \{C, \exists R.C, \forall R.(\exists R.C)\}$
R
z
$\mathcal{L}(z) = \{C\}$

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role


$\mathcal{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$
w
S
R
$\mathcal{L}(x) = \{C, (\neg C \sqcup \neg D), \neg D\}$
x
y
$\mathcal{L}(y) = \{C, \exists R.C, \forall R.(\exists R.C)\}$
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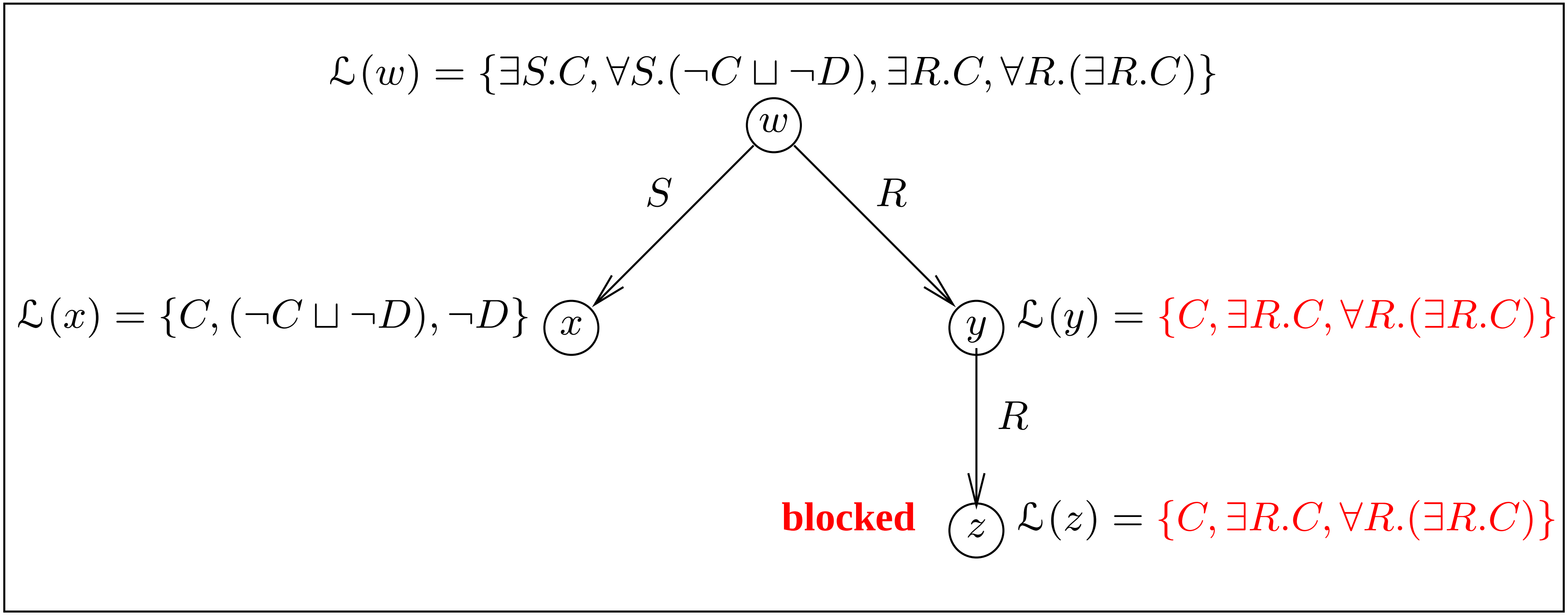
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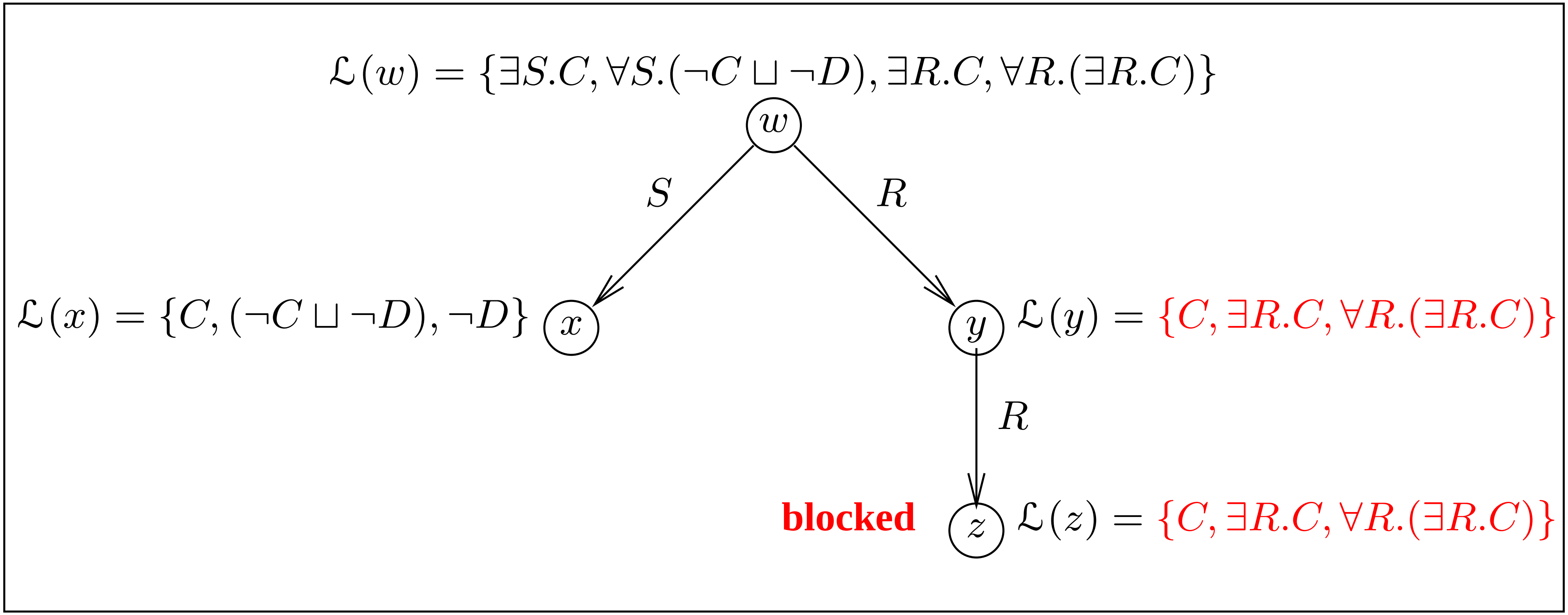

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Concept is **satisfiable**: $\mathbf{T}$ corresponds to **model**

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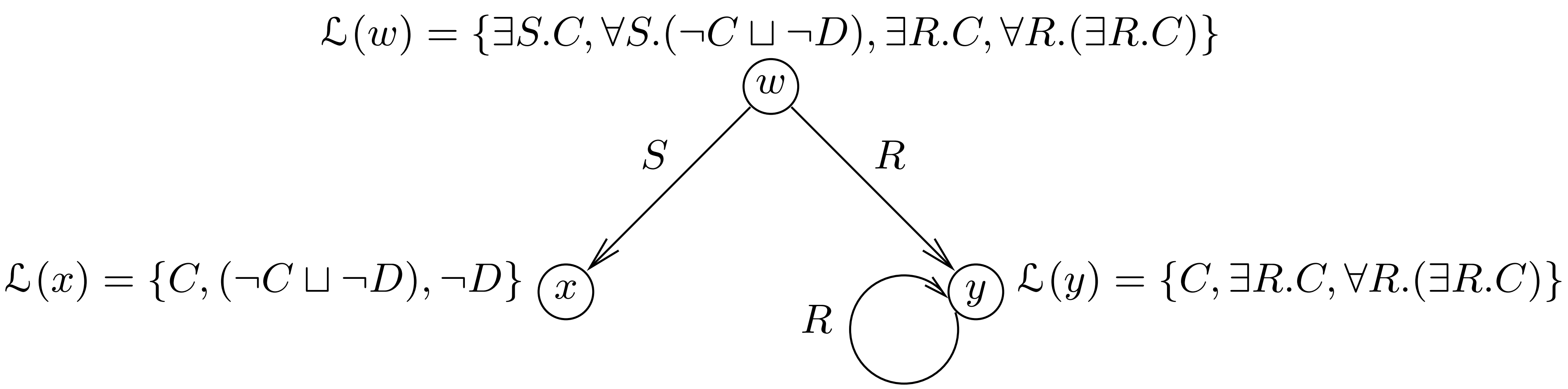

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Concept is **satisfiable**: $\mathbf{T}$ corresponds to **model**

Properties of our tableau algorithm for $\mathcal{ALC}$ with TBoxes

Lemma: Let $\mathcal{T}$ be a general $\mathcal{ALC}$-Tbox and C_0 an $\mathcal{ALC}$-concept. Then

1. the algorithm terminates when applied to $\mathcal{T}$ and C_0 and
2. the rules can be applied such that they generate a clash-free and complete completion tree iff C_0 is satisfiable w.r.t. $\mathcal{T}$.

Corollary:

1. Satisfiability of $\mathcal{ALC}$-concept w.r.t. TBoxes is decidable
2. $\mathcal{ALC}$ with TBoxes has the finite model property
3. $\mathcal{ALC}$ with TBoxes has the tree model property

Proof of the Lemma: Termination

(1) termination is, again, due to the following properties: let $n = |C_0| + |C_{\mathcal{T}}|$ and

$$\textbf{sub}(C_0, \mathcal{T}) = \textbf{sub}(C_0) \cup \bigcup_{C \dot{\sqsubseteq} D \in \mathcal{T}} \textbf{sub}(C) \cup \textbf{sub}(D)$$

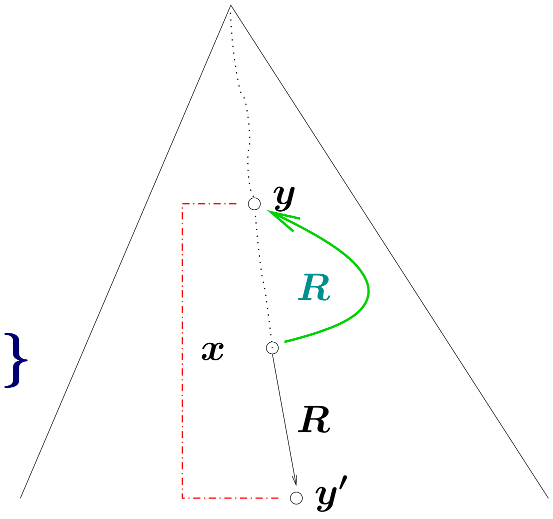
1. the c- tree is built in a **monotonic way**:
   each rule either extends node labels or adds a node (with a label)
2. node labels are restricted to subsets of **sub**$(C_0, \mathcal{T})$ and $\#\,$**sub**$(C_0, \mathcal{T}) \leq n$
3. the **breadth** of the c-tree is bounded by n:
   at most 1 successor per $\exists R.C \in$ **sub**$(C_0, \mathcal{T})$
4. the **depth** of the c-tree is bounded:
   on a path of length 2^n, blocking occurs, and thus it does not get longer

Important: in the presence of TBoxes, c-tree can be of **exponential** depth
whereas without TBoxes, depth was linearly bounded

Proof of the Lemma: Soundness

(2) let the algorithm stop with a complete and clash-free c-tree.
Again, from this, we define an interpretation:

$$\begin{aligned}
\Delta^{\mathcal{I}} &:= \{x \mid x \text{ is a node in } \mathcal{T}, x \text{ is not blocked}\}\\
A^{\mathcal{I}} &:= \{x \in \Delta^{\mathcal{I}} \mid A \in \mathcal{L}(x)\} \text{ for concept names } A\\
R^{\mathcal{I}} &:= \{\langle x, y\rangle \in \Delta^{\mathcal{I}^2} \mid \; y \text{ is an } R\text{-succ of } x \text{ in c-tree or}\\
&\qquad\qquad\qquad\qquad\quad\; y \text{ blocks an } R\text{-succ of } x \text{ in c-tree}\}
\end{aligned}$$



and show, by induction on the structure of concepts, for all $x \in \Delta^{\mathcal{I}}$, $D \in \mathsf{sub}(C_0, \mathcal{T})$:

$$D \in \mathcal{L}(x) \text{ implies } x \in D^{\mathcal{I}}.$$

This implies that $\mathcal{I}$ **is indeed a model of** C_0 **and** $\mathcal{T}$ because
(a) C_0 is in the label of the root node which cannot be blocked (!) and
(b) $\neg C \sqcup D$ is in the label of each node, for each $C \mathrel{\dot\sqsubseteq} D \in \mathcal{T}$

Proof of the Lemma: Completeness

(3) Let C_0 be satisfiable w.r.t. $\mathcal{T}$ and $\mathcal{I}$ a model of them with $a_0 \in C_0^{\mathcal{I}}$.

Use $\mathcal{I}$ to steer the application of the (only non-deterministic) $\sqcup$-rule:

Inductively define a total mapping π : nodes of completion tree $\longrightarrow \Delta^{\mathcal{I}}$, start with $\pi(x_0) = a_0$, and show that

each rule can be applied in such a way that $(*)$ is preserved

$$\begin{array}{ll} \text{if } C \in \mathcal{L}(x), \text{ then } \pi(x) \in C^{\mathcal{I}} & (*) \\ \text{if } y \text{ is an } R\text{-succ. of } x, \text{ then } \langle \pi(x), \pi(y) \rangle \in R^{\mathcal{I}} & \end{array}$$

- easy for $\sqcap$-, $\mathcal{T}$-, and $\forall$-rule,
- for $\exists$-rule, we need to extend π to the newly created R-successor
- for $\sqcup$-rule, if $C_1 \sqcup C_2 \in \mathcal{L}(x)$, $(*)$ implies that $\pi(x) \in (C_1 \sqcup C_2)^{\mathcal{I}}$
  $\rightsquigarrow$ we can choose C_i with $\pi(x) \in C_i^{\mathcal{I}}$ to add to $\mathcal{L}(x)$ and thus preserve $(*)$

$\rightsquigarrow$ easy to see: **$(*)$ implies that c-tree is clash-free**

Proof of the Lemma: Harvest

Look again at the model $\mathcal{I}$ constructed for a clash-free, complete c-tree:

$\mathcal{I}$ is

- **finite** because c-tree has finitely many nodes
- but it is **not a tree** if blocking occurs

Hence we get Corollary (2) for free from our proof:

C_0 is satisfiable

$\rightsquigarrow$ tableau algorithm stops with clash-free, complete c-tree

$\rightsquigarrow$ C_0 has a finite model.

To obtain Corollary (3), the tree model property, we must work a bit more:

$\rightsquigarrow$ build the model in a different way, "unravel" the c-tree into an infinite tree intuitively, instead of going to a blocked node, go to a copy of its blocking node

A tableau algorithm for $\mathcal{ALC}$ with general TBoxes: Summary

The tableau algorithm presented here

- **decides** satisfiability of $\mathcal{ALC}$-concepts w.r.t. TBoxes, and thus also
- **decides** subsumption of $\mathcal{ALC}$-concepts w.r.t. TBoxes
- uses **blocking** to ensure termination, and
- is **non-deterministic** due to the $\longrightarrow_{\sqcup}$-rule
- in the worst case, it builds a tree of depth exponential in the size of the input, and thus of double exponential size. Hence it runs in (worst case) 2NExpTime,
- can be implemented in various ways,
  - order/priorities of rules
  - data structure
  - etc.
- is amenable to optimisations – more on this next week

What next?

Next, we could

- discuss implementation issues for our tableau algorithms, e.g.,
  - datastructures,
  - more efficient (i.e., less strict) blocking conditions,
  - a good strategy for the order of rule applications,
  - how to “determinise” our non-deterministic algorithm: e.g., backtracking
  - etc.
- discuss other reasoning techniques for DLs
- analyse computational complexity of DLs
- further extend our tableau algorithm for more expressive DLs
  with one more expressive means

Naive Implementations

Problems include:

- ☞ **Space** usage
  - Storage required for tableaux datastructures
  - Rarely a serious problem in practice
  - But problems can arise with inverse roles and cyclical KBs
- ☞ **Time** usage
  - Search required due to non-deterministic expansion
  - **Serious** problem in practice
  - Mitigated by:
    - Careful **choice of algorithm**
    - Highly **optimised implementation**

Careful Choice of Algorithm

- ☞ **Transitive roles** instead of transitive closure
  - Deterministic expansion of $\exists R.C$, even when $R \in \mathbf{R}_+$
  - (Relatively) simple blocking conditions
  - Cycles **always** represent (part of) valid cyclical models
- ☞ **Direct algorithm**/implementation instead of encodings
  - GCI axioms can be used to “encode” additional operators/axioms
  - Powerful technique, particularly when used with FL closure
  - Can encode cardinality constraints, inverse roles, range/domain, . . .
    - E.g., $(\text{domain } R.C) \equiv \exists R.\top \sqsubseteq C$
  - (FL) encodings introduce (large numbers of) axioms
  - **BUT** even simple domain encoding is **disastrous** with large numbers of roles

Dependency Directed Backtracking

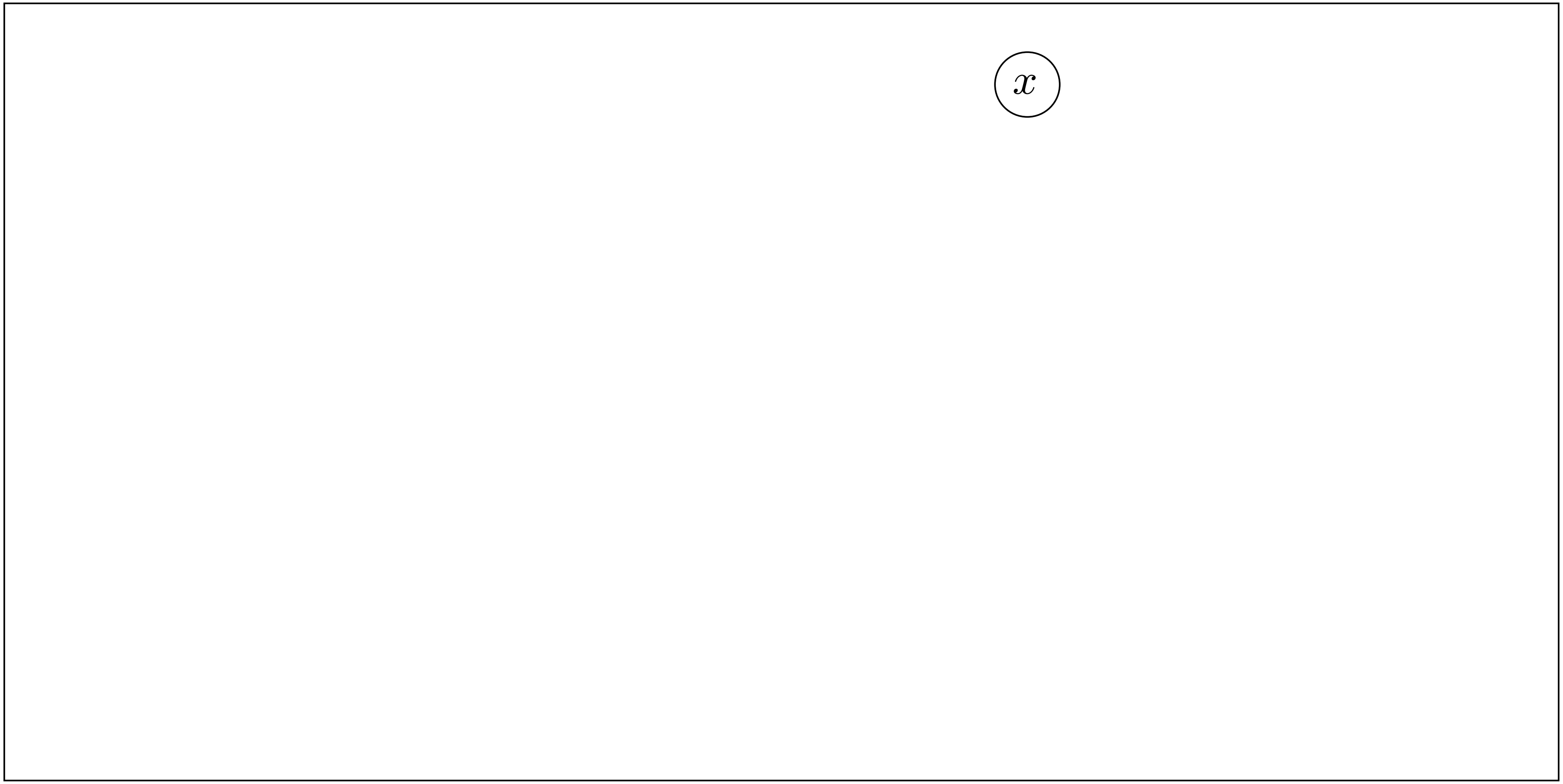
- ☞ Allows **rapid recovery** from bad branching choices
- ☞ Most commonly used technique is **backjumping**
  - Tag concepts introduced at **branch points** (e.g., when expanding disjunctions)
  - Expansion rules combine and **propagate tags**
  - On discovering a clash, **identify** most recently introduced concepts involved
  - **Jump back** to relevant branch points **without exploring** alternative branches
  - Effect is to **prune** away part of the search space
- ☞ **Highly effective** — essential for usable system
  - E.g., GALEN KB, 30s (with) $\longrightarrow$ months++ (without)

Backjumping

E.g., if $\exists R.\neg A \sqcap \forall R.(A \sqcap B) \sqcap (C_1 \sqcup D_1) \sqcap \ldots \sqcap (C_n \sqcup D_n) \subseteq \mathcal{L}(x)$

Backjumping

E.g., if $\exists R.\neg A \sqcap \forall R.(A \sqcap B) \sqcap (C_1 \sqcup D_1) \sqcap \ldots \sqcap (C_n \sqcup D_n) \subseteq \mathcal{L}(x)$



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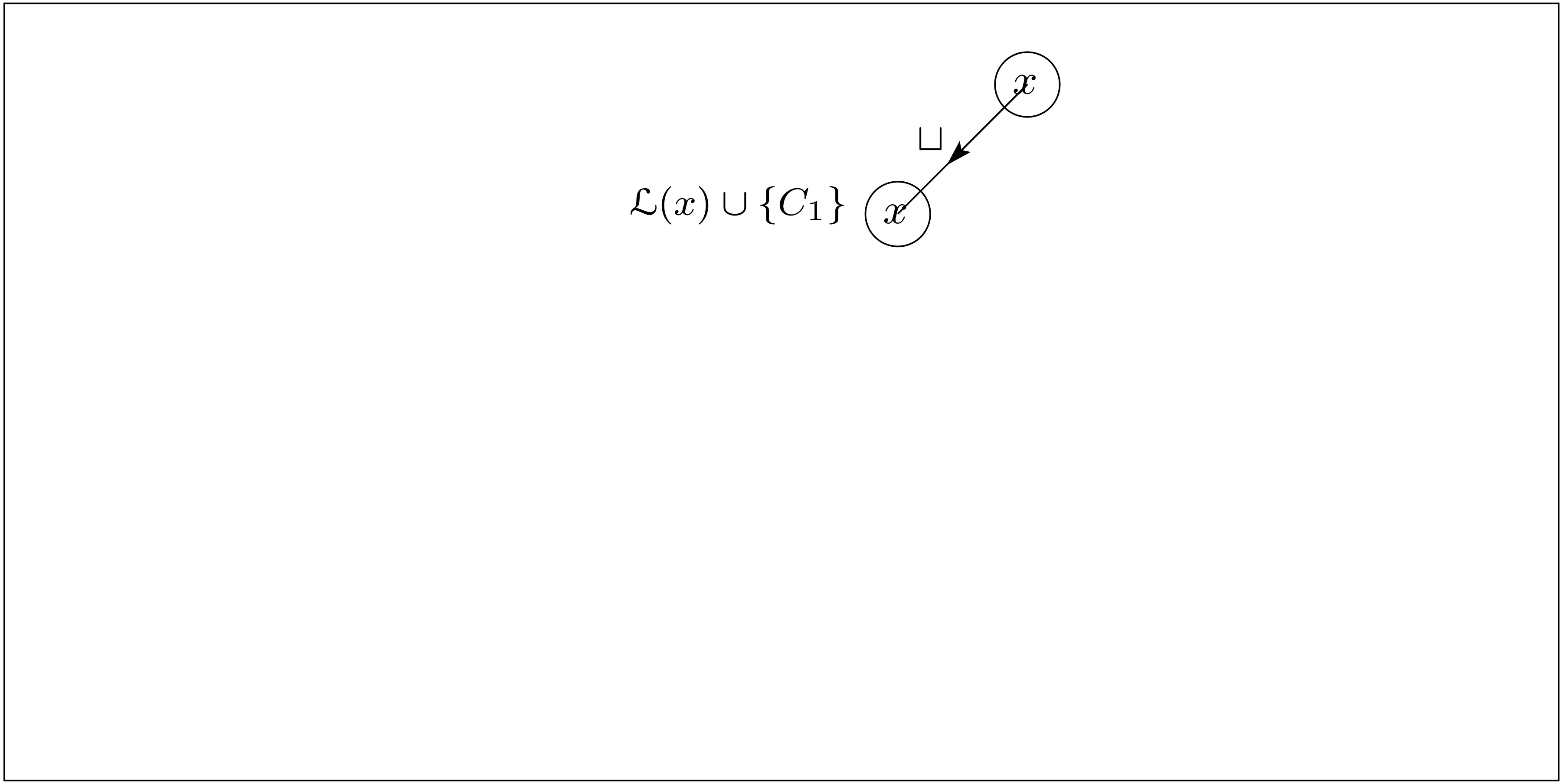

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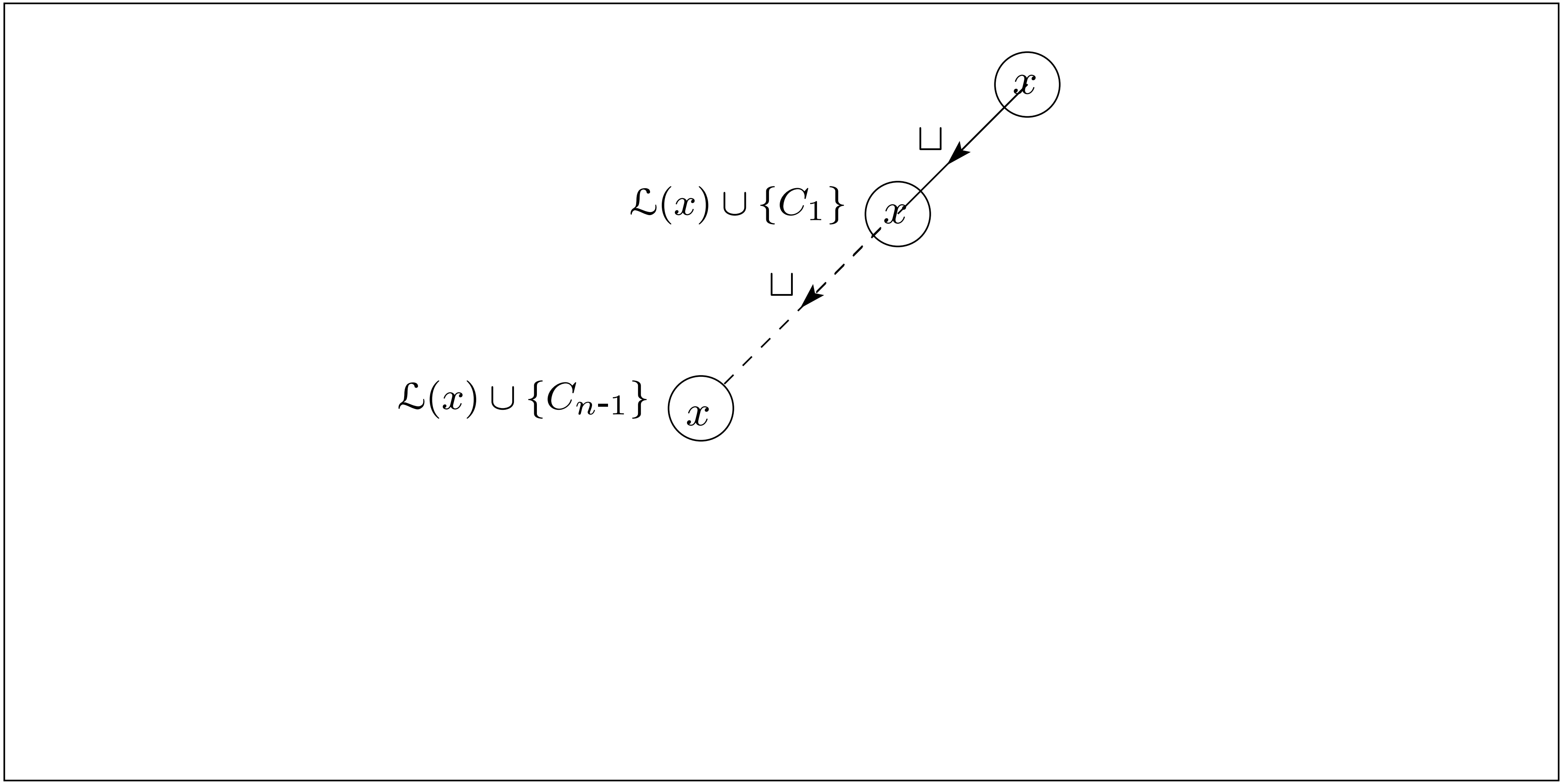
x
$\sqcup$
$\mathcal{L}(x) \cup \{C_1\}$
x
$\sqcup$
$\mathcal{L}(x) \cup \{C_{n\text{-}1}\}$
x

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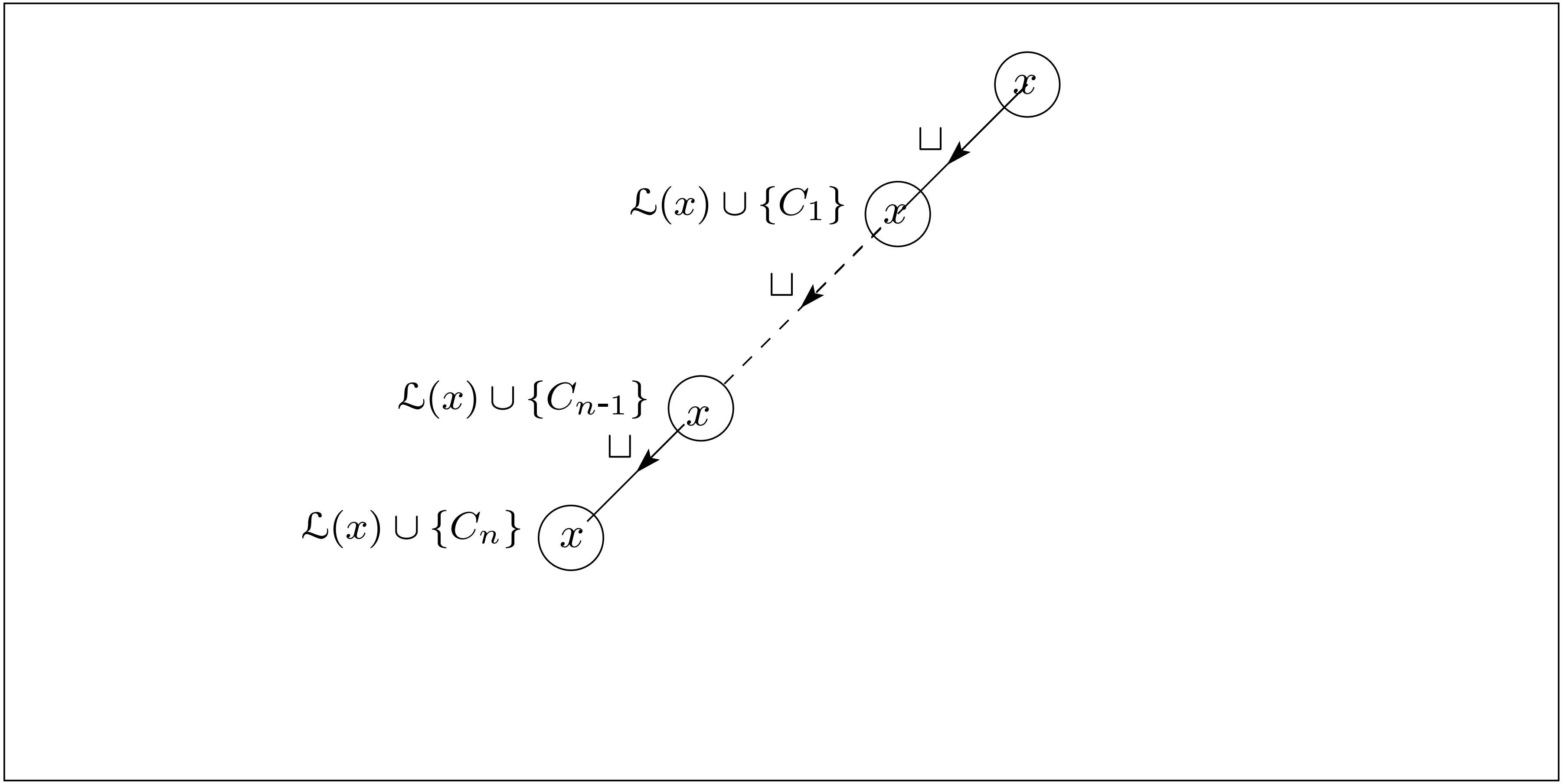

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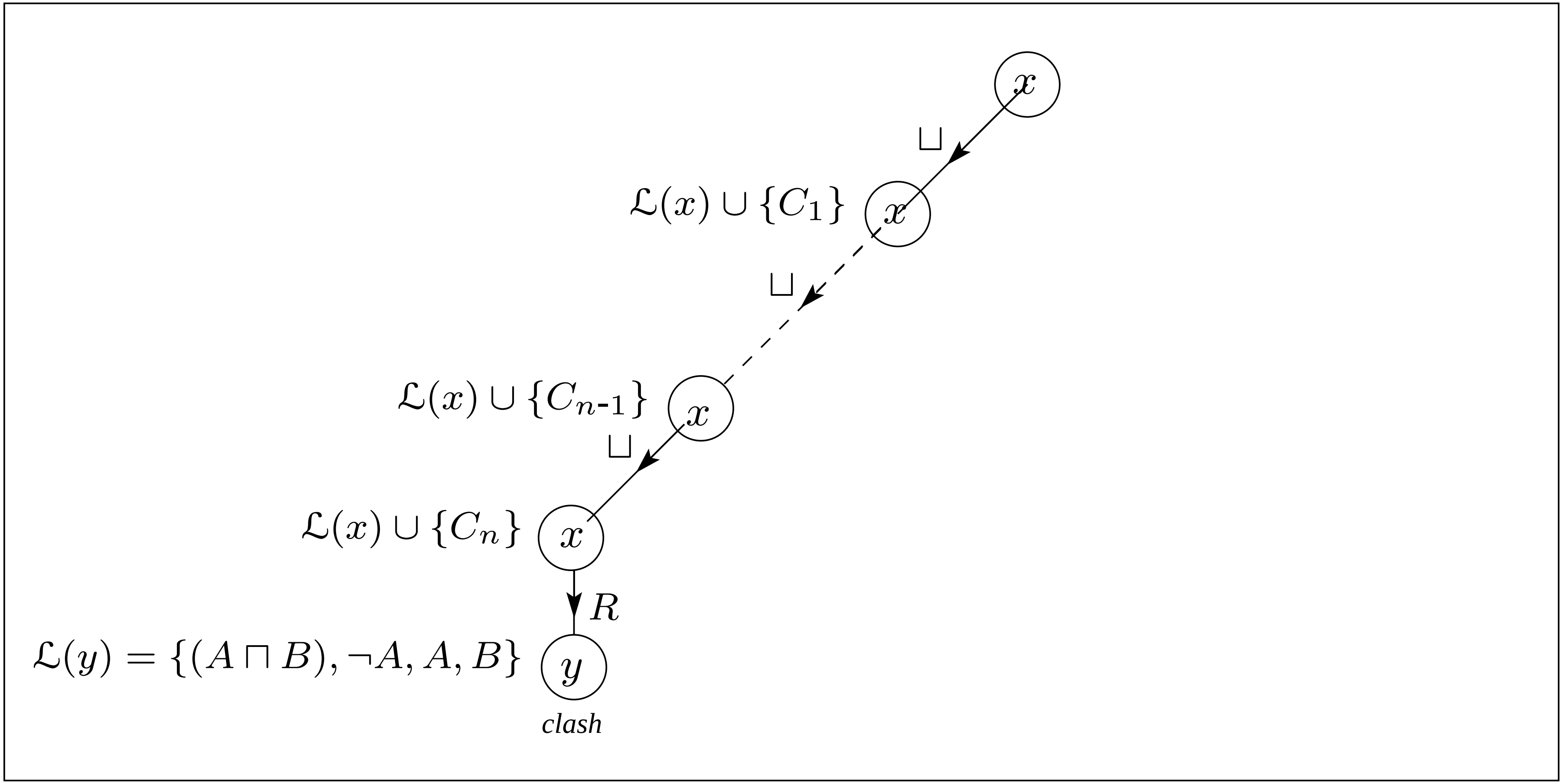

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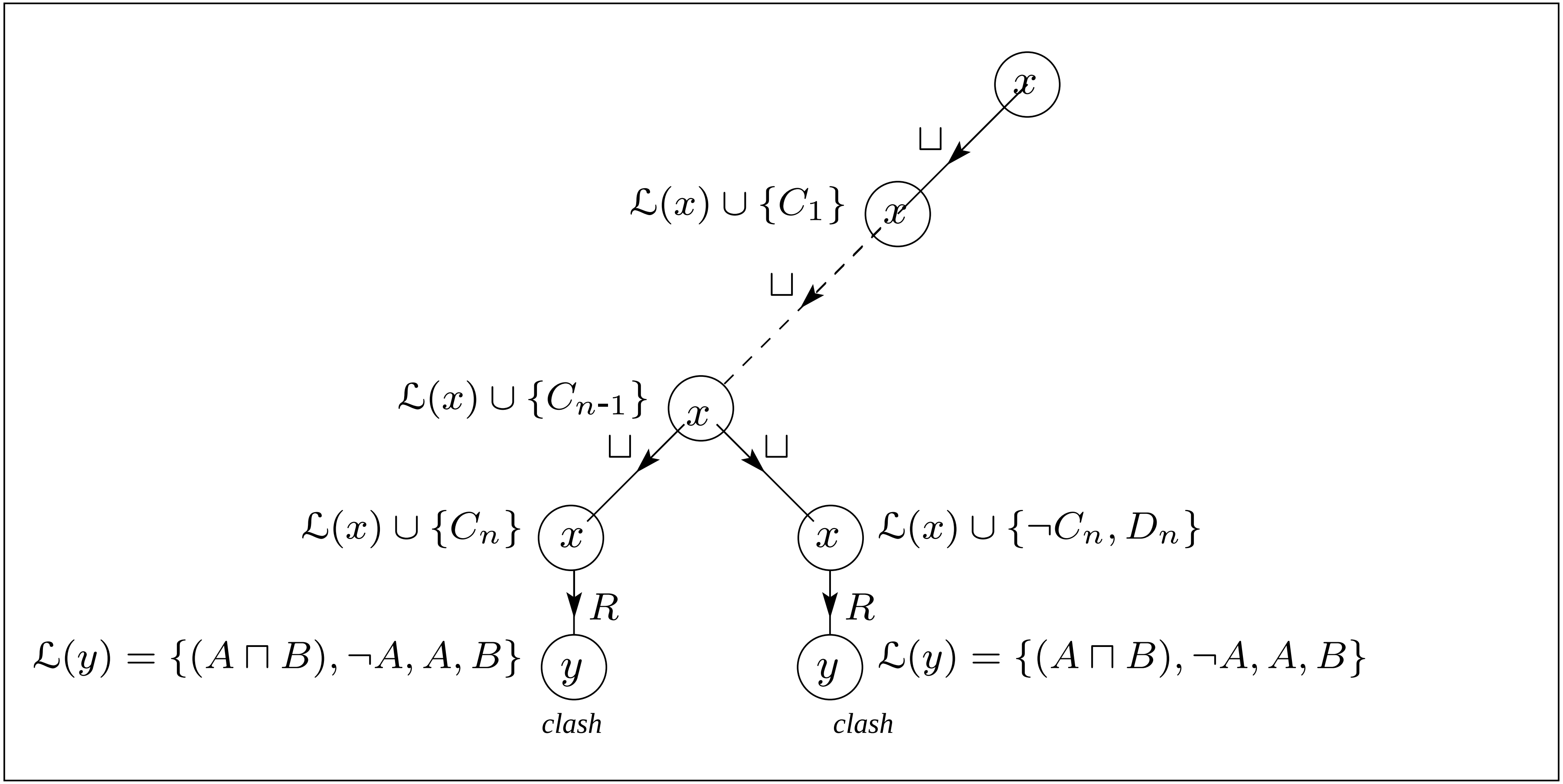
x
$\sqcup$
$\mathcal{L}(x) \cup \{C_n\}$
x
R
$\mathcal{L}(y) = \{(A \sqcap B), \neg A, A, B\}$
y
clash

Backjumping

E.g., if $\exists R.\neg A \sqcap \forall R.(A \sqcap B) \sqcap (C_1 \sqcup D_1) \sqcap \ldots \sqcap (C_n \sqcup D_n) \subseteq \mathcal{L}(x)$



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$\sqcup$
$\mathcal{L}(x) \cup \{C_1\}$
x
$\sqcup$
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x
$\sqcup$
$\sqcup$
$\mathcal{L}(x) \cup \{C_n\}$
x
x
$\mathcal{L}(x) \cup \{\neg C_n, D_n\}$
R
R
$\mathcal{L}(y) = \{(A \sqcap B), \neg A, A, B\}$
y
y
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clash
clash

Backjumping

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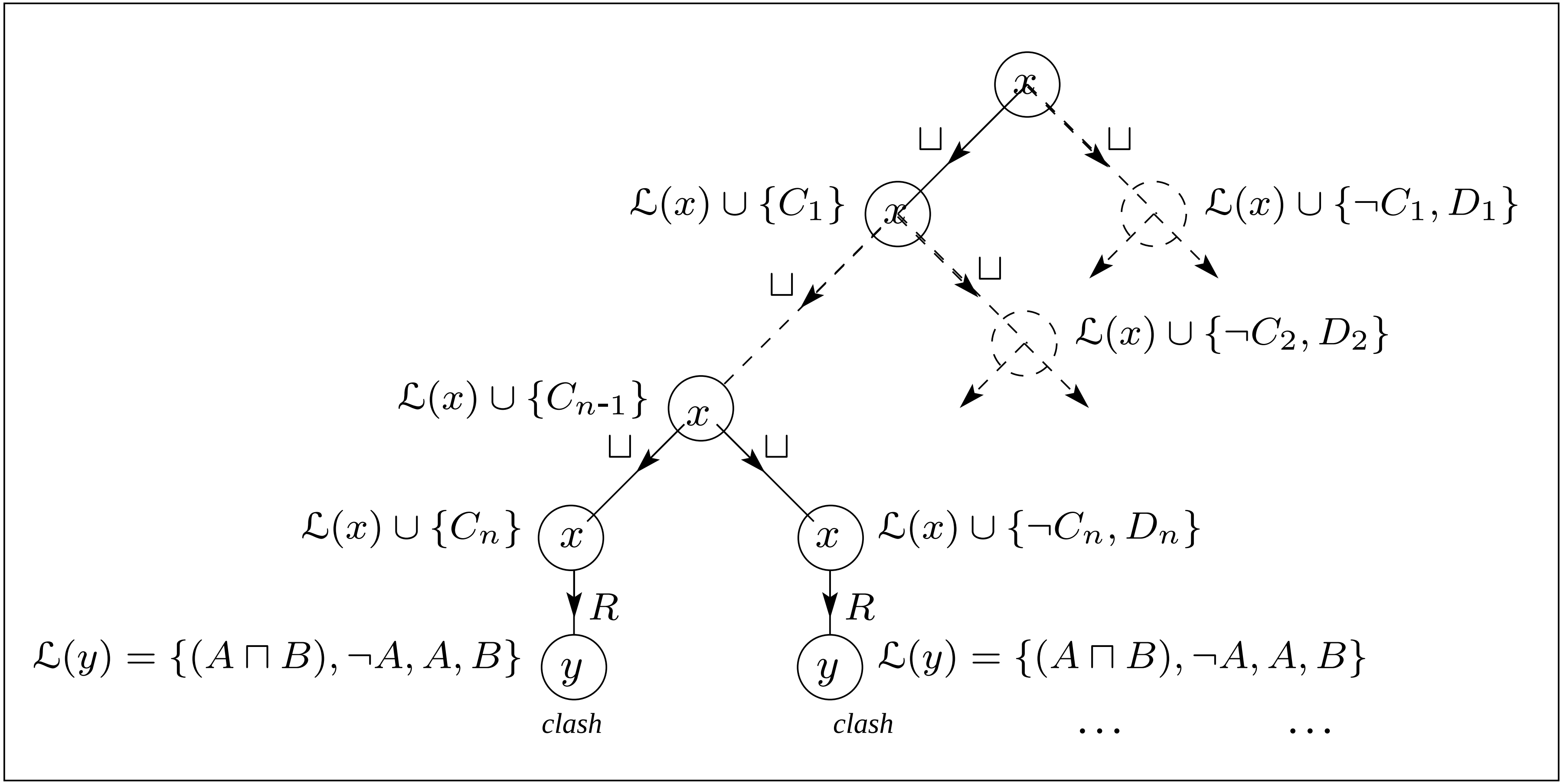

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x
x
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R
R
$\mathcal{L}(y) = \{(A \sqcap B), \neg A, A, B\}$
y
y
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clash
clash
...
...

Backjumping

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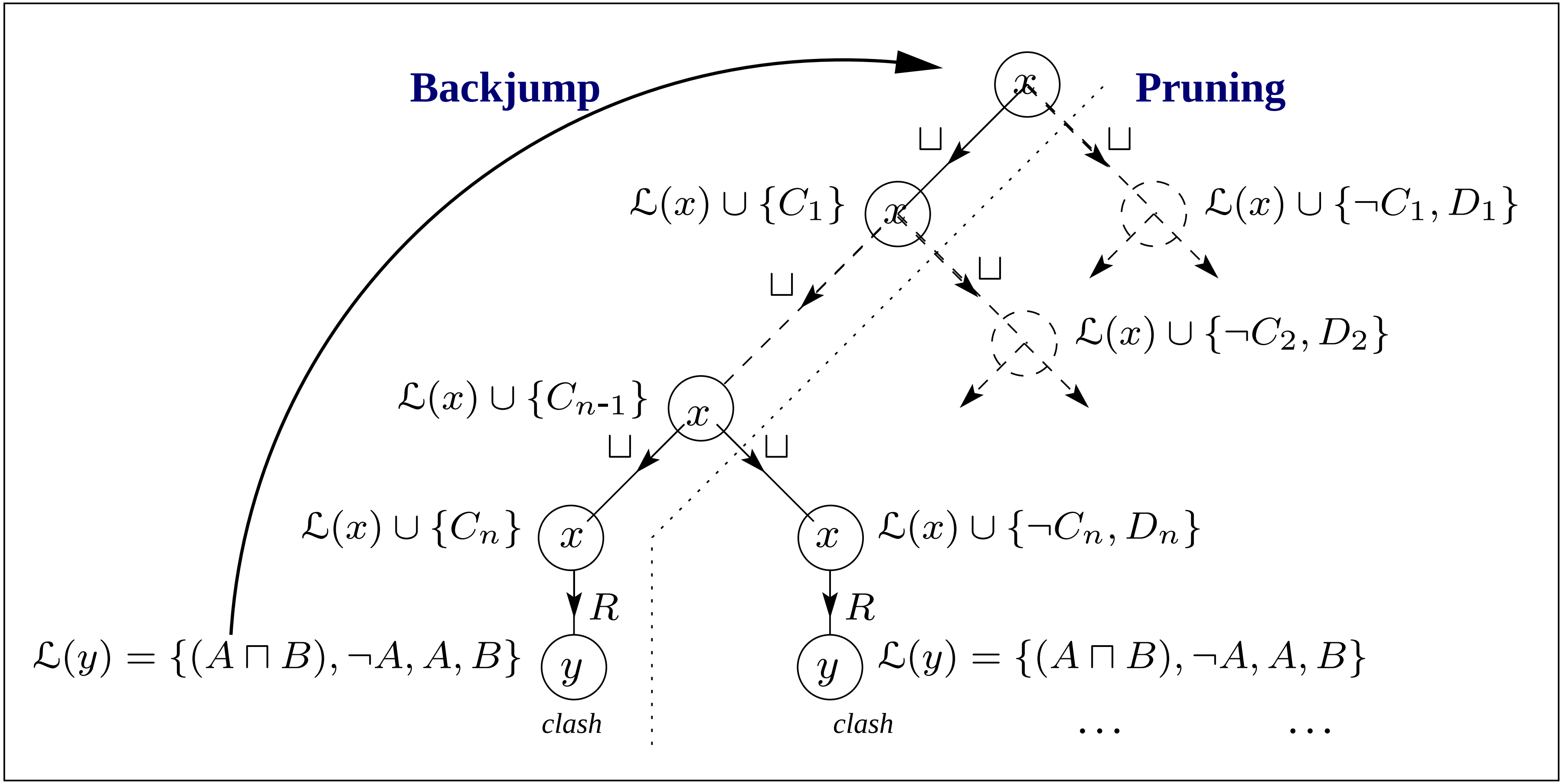

Backjump
Pruning
x
$\sqcup$
$\sqcup$
$\mathcal{L}(x) \cup \{C_1\}$
$\mathcal{L}(x) \cup \{\neg C_1, D_1\}$
$\sqcup$
$\sqcup$
$\mathcal{L}(x) \cup \{\neg C_2, D_2\}$
$\mathcal{L}(x) \cup \{C_{n\text{-}1}\}$
$\sqcup$
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$\mathcal{L}(x) \cup \{C_n\}$
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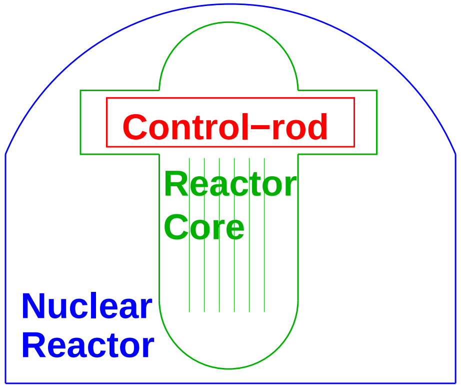
R
R
$\mathcal{L}(y) = \{(A \sqcap B), \neg A, A, B\}$
y
$\mathcal{L}(y) = \{(A \sqcap B), \neg A, A, B\}$
clash
clash
...
...

Inverse Roles



Consider the following TBox

$$\begin{aligned}\texttt{Control-rod} &\mathrel{\dot\sqsubseteq} \texttt{Device} \sqcap \exists\texttt{part-of}.\texttt{Reactor-core}\\ \texttt{Reactor-core} &\mathrel{\dot\sqsubseteq} \texttt{Device} \sqcap \exists\texttt{has-part}.\texttt{Control-rod} \sqcap\\ &\qquad \exists\texttt{part-of}.\texttt{N-reactor},\\ \texttt{Reactor-core} \sqcap \exists\texttt{has\_part}.\texttt{Faulty} &\mathrel{\dot\sqsubseteq} \texttt{Dangerous},\end{aligned}$$

Now, w.r.t. such a TBox, we find that

$\texttt{Control\_rod} \sqcap \texttt{Faulty}$ should be subsumed by $\exists\texttt{part-of}.\texttt{Dangerous}$

But this is not true: no **interaction** between part-of and has-part!

$\rightsquigarrow$ also allow for $\exists R^-.C$ and $\forall R^-.C$, where $(R^-)^\mathcal{I} = \{\langle y, x\rangle \mid \langle x, y\rangle \in R^\mathcal{I}\}$

A tableau algorithm for $\mathcal{ALCI}$ with general TBoxes

$\mathcal{ALCI}$ is the extension of $\mathcal{ALC}$ with **inverse roles** R^- in the place of role names:

$$(R^-)^\mathcal{I} := \{\langle y, x\rangle \mid \langle x, y\rangle \in R^\mathcal{I}\}$$

Example: does $\forall$parent.$\forall$child.Blond $\sqsubseteq$ Blond w.r.t. $\{\top \mathrel{\dot\sqsubseteq} \exists$parent.$\top\}$?

does $\forall$parent.$\forall$parent$^-$.Blond $\sqsubseteq$ Blond w.r.t. $\{\top \mathrel{\dot\sqsubseteq} \exists$parent.$\top\}$?

Example: is $C_0 = \exists R.\exists S.\exists T.A$ satisf. w.r.t. $\{ C \mathrel{\dot\sqsubseteq} \exists R.C \sqcap \forall R.B$
$\top \mathrel{\dot\sqsubseteq} \forall T^-.\forall S^-.\forall R^-.C\}$?

Clear: inverse roles $\rightsquigarrow$ tableau algorithm must reason *up and down* edges

A tableau algorithm for $\mathcal{ALCI}$ with general TBoxes

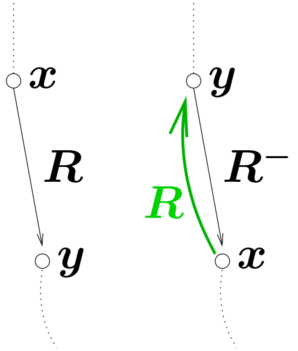
Modifications necessary to handle inverse roles:

① extend edge labels in c-trees to inverse roles,

② call y an R-**neighbour** of x if either

y is an R-successor of x or

x is an R^- successor of y,



③ substitute "R-successor" in the $\forall$- and $\exists$-rule with "R-neighbour"

still create an R-**successor** of x if no R-neighbour exists for $\exists R.C \in \mathfrak{L}(x)$

R^--**successor** of x if no R^--neighbour exists for an $\exists R^-.C \in \mathfrak{L}(x)$

A tableau algorithm for $\mathcal{ALCI}$ with general TBoxes

$\sqcap$-rule: if $C_1 \sqcap C_2 \in \mathcal{L}(x)$, $\{C_1, C_2\} \not\subseteq \mathcal{L}(x)$, and x is not blocked
then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C_1, C_2\}$

$\sqcup$-rule: if $C_1 \sqcup C_2 \in \mathcal{L}(x)$, $\{C_1, C_2\} \cap \mathcal{L}(x) = \emptyset$, and x is not blocked
then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C\}$ for some $C \in \{C_1, C_2\}$

$\exists$-rule: if $\exists S.C \in \mathcal{L}(x)$, x has no S-neighbour y with $C \in \mathcal{L}(y)$,
and x is not blocked
then create a new node y with $\mathcal{L}(\langle x, y\rangle) = \{S\}$ and $\mathcal{L}(y) = \{C\}$

$\forall$-rule: if $\forall S.C \in \mathcal{L}(x)$, there is an S-neighbour y of x with $C \notin \mathcal{L}(y)$
and x is not indirectly blocked
then set $\mathcal{L}(y) = \mathcal{L}(y) \cup \{C\}$

$\mathcal{T}$-rule: if $C_1 \dot{\sqsubseteq} C_2 \in \mathcal{T}$, NNF$(\neg C_1 \sqcup C_2) \notin \mathcal{L}(x)$
and x is not blocked
then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{\text{NNF}(\neg C_1 \sqcup C_2)\}$

A tableau algorithm for $\mathcal{ALCI}$ with general TBoxes

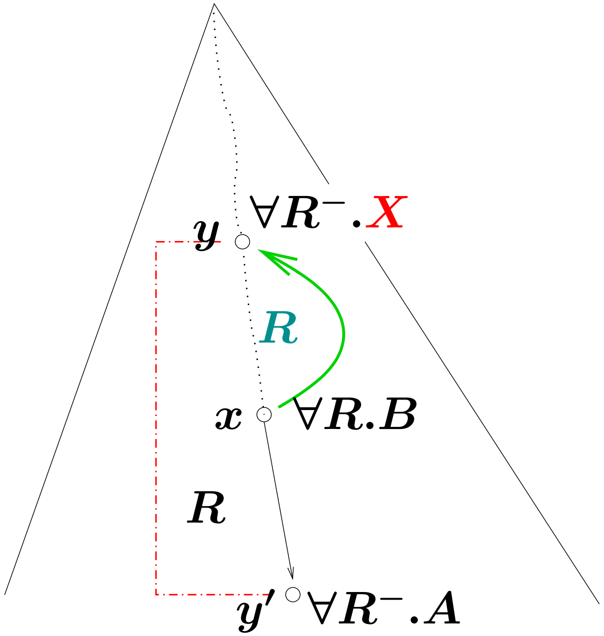
Example: is A satisfiable w.r.t. $\{A \mathrel{\dot\sqsubseteq} \exists R^-.A \sqcap \forall R.(\neg A \sqcup \exists S.B)\}$?

Example: is $\exists R.B$ satisfiable w.r.t. $\{B \mathrel{\dot\sqsubseteq} \exists R.B \sqcap \forall R^-.\forall R^-.\bot\}$?

Problem: algorithm returns *"satisfiable"* for unsatisfiable input ⇝ **incorrect!**

Reason: blocking condition $\mathcal{L}(y') \subseteq \mathcal{L}(y)$ is too loose: universal value restrictions from blocking node may be violated

Solution: tighten blocking condition to $\mathcal{L}(y') = \mathcal{L}(y)$



A tableau algorithm for $\mathcal{ALCI}$ with general TBoxes

④ A node x is **directly blocked** if it has an ancestor y with $\mathcal{L}(x) = \mathcal{L}(y)$.

Lemma: Let $\mathcal{T}$ be a general $\mathcal{ALCI}$-Tbox and C_0 an $\mathcal{ALCI}$-concept. Then

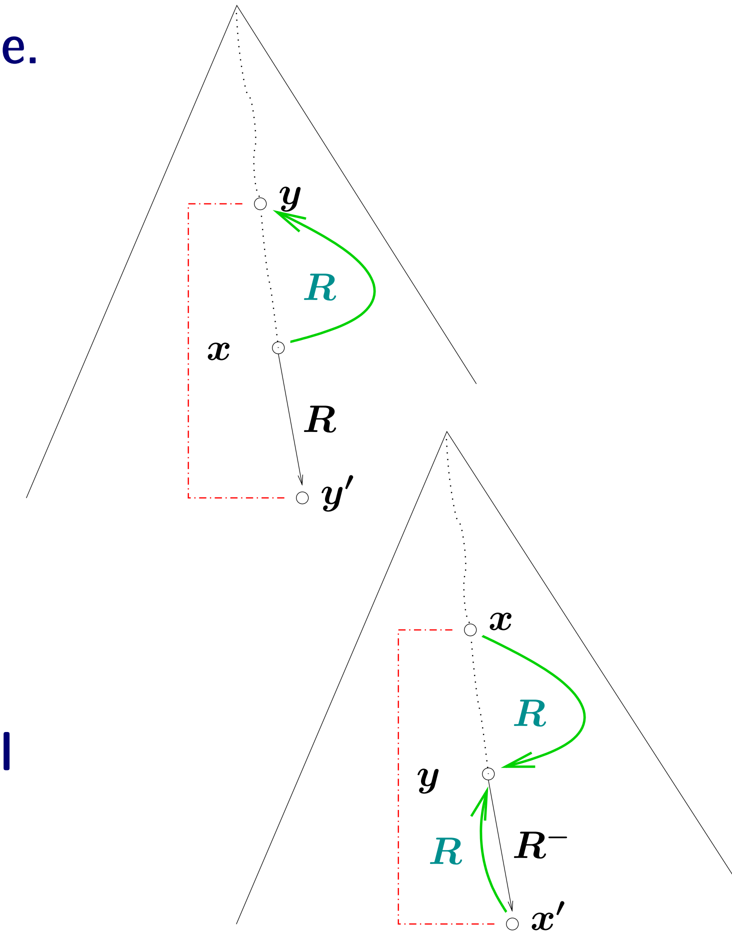
1. the algorithm terminates when applied to $\mathcal{T}$ and C_0,
2. the rules can be applied such that they generate a clash-free and complete completion tree iff C_0 is satisfiable w.r.t. $\mathcal{T}$.

Proof: (1) termination is identical to the $\mathcal{ALC}$ case.

Proof of the Lemma: Soundness

(2) let the algorithm stop with a complete and clash-free c-tree.
Again, from this, we define an interpretation:

$$
\begin{array}{lcl}
\Delta^{\mathcal{I}} & := & \{x \mid x \text{ is a node in } \mathcal{T}, x \text{ is not blocked}\} \\
A^{\mathcal{I}} & := & \{x \in \Delta^{\mathcal{I}} \mid A \in \mathfrak{L}(x)\} \text{ for concept names } A \\
R^{\mathcal{I}} & := & \{\langle x, y\rangle \in \Delta^{\mathcal{I}^2} \mid \begin{array}{l} y \text{ is an } R\text{-succ of } x \text{ or} \\ y \text{ blocks an } R\text{-succ of } x \text{ or} \\ x \text{ is an } R^-\text{-succ of } y \text{ or} \\ x \text{ blocks an } R^-\text{-succ of } y \;\} \end{array}
\end{array}
$$



and show, by induction on the structure of concepts, for all $x \in \Delta^{\mathcal{I}}$, $D \in \mathsf{sub}(C_0, \mathcal{T})$:

$$D \in \mathfrak{L}(x) \text{ implies } x \in D^{\mathcal{I}}.$$

As for $\mathcal{ALC}$, this implies that $\mathcal{I}$ **is indeed a model of** C_0 **and** $\mathcal{T}$

Proof of the Lemma: Completeness

(3) completely identical to the $\mathcal{ALC}$ case...

That's it!

I hope you got an idea of how we can

- build tableau algorithms for description logics and
- see that they do indeed what we want them to do, i.e., decide satisfiability

Research Challenges

Challenges

- ☞ **Increased expressive power**
  - Existing DL systems implement (at most) $\mathcal{SHIQ}$
  - OWL extends $\mathcal{SHIQ}$ with datatypes and nominals
- ☞ **Scalability**
  - Very large KBs
  - Reasoning with (very large numbers of) individuals
- ☞ **Other reasoning tasks**
  - Querying
  - Matching
  - Least common subsumer
  - . . .
- ☞ **Tools and Infrastructure**
  - Support for large scale ontological engineering and deployment

Increased Expressive Power: Datatypes

- **OWL** has simple form of datatypes
  - Unary predicates plus disjoint object-class/datatype domains
- Well understood **theoretically**
  - Existing work on **concrete domains** [Baader & Hanschke, Lutz]
  - Algorithm already known for $\mathcal{SHOQ}(\mathbf{D})$ [Horrocks & Sattler]
  - Can use **hybrid reasoning** (DL reasoner + datatype “oracle”)
- May be **practically** challenging
  - All XMLS datatypes supported (?)
- Already seeing some (partial) **implementations**
  - Cerebra system (Network Inference), Racer system (Hamburg)

Increased Expressive Power: Nominals

- ☞ OWL **oneOf** constructor equivalent to hybrid logic **nominals**
  - Extensionally defined concepts, e.g., EU $\equiv \{\text{France}, \text{Italy}, \ldots\}$
- ☞ Theoretically **very challenging**
  - Resulting logic has known **high complexity** (NExpTime)
  - No known “practical” algorithm
  - Not obvious how to extend tableaux techniques in this direction
    - Loss of tree model property
    - Spy-points: $\top \sqsubseteq \exists R.\{Spy\}$
    - Finite domains: $\{Spy\} \sqsubseteq \leqslant n R^-$
- ☞ **Standard solution** is weaker semantics for nominals
  - Treat nominals as (disjoint) primitive classes
  - Loss of completeness/soundness

Increased Expressive Power: Extensions

- OWL **not expressive enough** for all applications
- Extensions **wish list** includes:
  - Feature chain (path) agreement, e.g., output of component of composite process equals input of subsequent process
  - Complex roles/role inclusions, e.g., a city located in part of a country is located in that country
  - Rules—proposal(s) already exist for “datalog/LP style rules”
  - Temporal and spatial reasoning
  - . . .
- May be impossible/undesirable to resist such extensions
- Extended language sure to be **undecidable**
- How can extensions best be **integrated** with OWL?
- How can reasoners be developed/adapted for extended languages
  - Some existing work on language **fusions** and **hybrid** reasoners

Scalability

- Reasoning **hard** (ExpTime) even without nominals (i.e., $\mathcal{SHIQ}$)
- Web ontologies may grow **very large**
- Good **empirical evidence** of scalability/tractability for DL systems
  - E.g., 5,000 (complex) classes; 100,000+ (simple) classes
- But evidence mostly w.r.t. $\mathcal{SHF}$ (no inverse)
- **Problems** can arise when $\mathcal{SHF}$ extended to $\mathcal{SHIQ}$
  - Important **optimisations** no longer (fully) work
- Reasoning with **individuals**
  - **Deployment** of web ontologies will mean reasoning with (possibly very large numbers of) individuals/tuples
  - Unlikely that standard **Abox** techniques will be able to cope

Performance Solutions (Maybe)

- Excessive **memory usage**
  - Problem exacerbated by over-cautious double blocking condition (e.g., root node can never block)
  - Promising results from more precise blocking condition [Sattler & Horrocks]
- **Qualified number restrictions**
  - Problem exacerbated by naive expansion rules
  - Promising results from optimised expansion using Algebraic Methods [Haarslev & Möller]
- **Caching** and merging
  - Can still work in some situations (work in progress)
- Reasoning with **very large KBs**
  - DL systems shown to work with ≈100k concept KB [Haarslev & Möller]
  - But KB only exploited small part of DL language

Other Reasoning Tasks

- ☞ **Querying**
  - Retrieval and instantiation wont be sufficient
  - Minimum requirement will be **DB style query language**
  - May also need “what can I say about x?” style of query
- ☞ **Explanation**
  - To support ontology design
  - Justifications and proofs (e.g., of query results)
- ☞ “**Non-Standard Inferences**”, e.g., LCS, matching
  - To support ontology integration
  - To support “bottom up” design of ontologies

Summary

- **Description Logics** are family of logical KR formalisms
- **Applications** of DLs include DataBases and **Semantic Web**
  - Ontologies will provide vocabulary for semantic markup
  - OWL web ontology language based on $\mathcal{SHIQ}$ DL
  - Set to become W3C standard (OWL) & already widely adopted
  - Use of DL provides formal foundations and reasoning support
- **DL Reasoning** based on tableau algorithms
- **Highly Optimised** implementations used in DL systems
- **Challenges** remain
  - Reasoning with full OWL language
  - (Convincing) demonstration(s) of scalability
  - New reasoning tasks
  - Development of (high quality) tools and infrastructure

Resources

Slides from this talk

`http://www.cs.man.ac.uk/~horrocks/Slides/Innsbruck-tutorial/`

FaCT system (open source)

`http://www.cs.man.ac.uk/FaCT/`

OilEd (open source)

`http://oiled.man.ac.uk/`

OIL

`http://www.ontoknowledge.org/oil/`

W3C Web-Ontology (WebOnt) working group (OWL)

`http://www.w3.org/2001/sw/WebOnt/`

DL Handbook, Cambridge University Press

`http://books.cambridge.org/0521781760.htm`

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