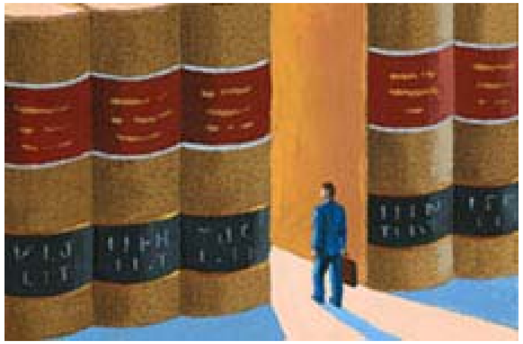
E. Description Logics



This section is based on material from

• Ian Horrocks: http://www.cs.man.ac.uk/~horrocks/Teaching/cs646/

Description Logics

- OWL DL ist äquivalent zur Beschreibungslogik $\mathcal{SHOIN}(\mathbf{D}_n)$. Auf letzterer basiert also die Semantik von OWL DL.
- Unter Beschreibungslogiken (Description Logics) versteht man eine Familie von Teilsprachen der Prädikatenlogik 1. Stufe, die entscheidbar sind.
- $\mathcal{SHOIN}(\mathbf{D}_n)$ ist eine relativ komplexe Beschreibungslogik.
- Um einen ersten Einblick in das Prinzip der Beschreibungslogiken zu erhalten, werfen wir zum Abschluss der Vorlesung einen Blick auf etwas abgespeckte Fassungen.

Literatur:

- D. Nardi, R. J. Brachman. An Introduction to Description Logics. In: F. Baader, D. Calvanese, D.L. McGuinness, D. Nardi, P.F. Patel-Schneider (eds.): Description Logic Handbook, Cambridge University Press, 2002, 5-44.
- F. Baader, W. Nutt: Basic Description Logics. In: Description Logic Handbook, 47-100.
- Ian Horrocks, Peter F. Patel-Schneider and Frank van Harmelen. From SHIQ and RDF to OWL: The making of a web ontology language. http://www.cs.man.ac.uk/%7Ehorrocks/Publications/download/2003/HoPH03a.pdf

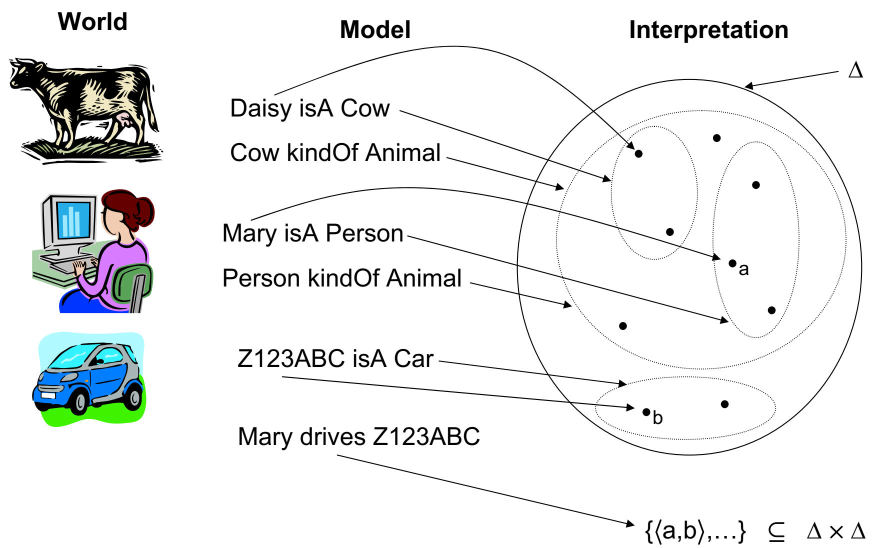
Aside: Semantics and Model Theories

- Ontology/KR languages aim to model (part of) world
- Terms in language correspond to entities in world
- Meaning given by, e.g.:
  - Mapping to another formalism, such as FOL, with own well defined semantics
  - or a bespoke Model Theory (MT)
- MT defines relationship between syntax and *interpretations*
  - Can be many interpretations (models) of one piece of syntax
  - Models supposed to be analogue of (part of) world
    - **E.g., elements of model correspond to objects in world**
  - Formal relationship between syntax and models
    - **Structure of models reflect relationships specified in syntax**
  - Inference (e.g., subsumption) defined in terms of MT
    - **E.g., $\mathcal{T} \models A \sqsubseteq B$ iff in every model of $\mathcal{T}$, ext(A) $\subseteq$ ext(B)**

Aside: Set Based Model Theory

- Many logics (including standard First Order Logic) use a model theory based on Zermelo-Frankel set theory
- The domain of discourse (i.e., the part of the world being modelled) is represented as a set (often refered as Δ)
- Objects in the world are interpreted as elements of Δ
  - Classes/concepts (unary predicates) are subsets of Δ
  - Properties/roles (binary predicates) are subsets of $\Delta \times \Delta$ (i.e., Δ^2)
  - Ternary predicates are subsets of Δ^3 etc.
- The sub-class relationship between classes can be interpreted as set inclusion
- Doesn't work for RDF, because in RDF a class (set) can be a member (element) of another class (set)
  - In Z-F set theory, elements of classes are atomic (no structure)

Aside: Set Based Model Theory Example



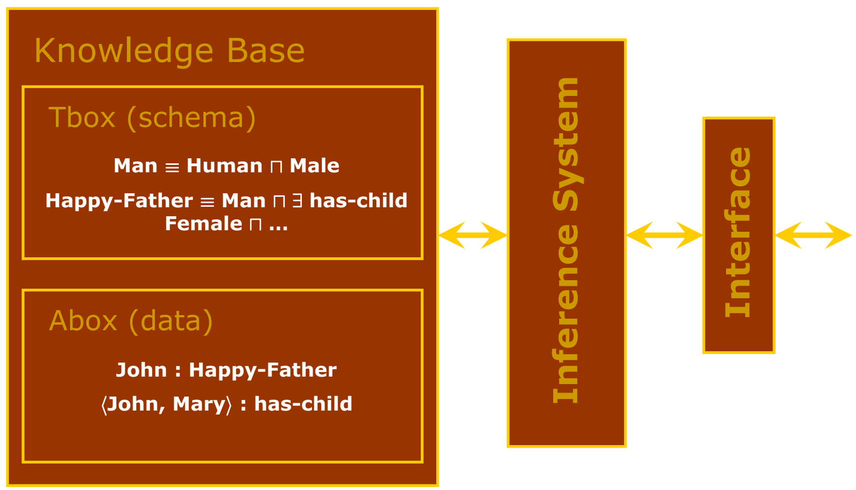
Aside: Set Based Model Theory Example

- Formally, the vocabulary is the set of names we use in our model of (part of) the world
  - {Daisy, Cow, Animal, Mary, Person, Z123ABC, Car, drives, …}
- An interpretation $\mathcal{I}$ is a tuple $\langle \Delta, \cdot^{\mathcal{I}} \rangle$
  - Δ is the domain (a set)
  - $\cdot^{\mathcal{I}}$ is a mapping that maps
    - Names of objects to elements of Δ
    - Names of unary predicates (classes/concepts) to subsets of Δ
    - Names of binary predicates (properties/roles) to subsets of $\Delta \times \Delta$
    - And so on for higher arity predicates (if any)

What Are Description Logics?

- **A family of logic based Knowledge Representation formalisms**
  - **Descendants of** semantic networks **and** KL-ONE
  - **Describe domain in terms of** concepts **(classes),** roles **(relationships) and** individuals
- **Distinguished by:**
  - Formal semantics **(typically model theoretic)**
    - Decidable fragments of FOL
    - Closely related to Propositional Modal & Dynamic Logics
  - **Provision of** inference services
    - Sound and complete decision procedures for key problems
    - Implemented systems (highly optimised)

DL Architecture



Short History of Description Logics

Phase 1:

- **Incomplete systems (Back, Classic, Loom, . . .)**
- **Based on** structural algorithms

Phase 2:

- **Development of** tableau algorithms **and** complexity results
- **Tableau-based systems for** Pspace **logics (e.g., Kris, Crack)**
- **Investigation of optimisation techniques**

Phase 3:

- **Tableau algorithms for** very expressive **DLs**
- Highly optimised **tableau systems for** ExpTime **logics (e.g., FaCT, DLP, Racer)**
- **Relationship to modal logic and decidable fragments of FOL**

Latest Developments

Phase 4:

- **Mature** implementations
- **Mainstream** applications **and Tools**
  - **Databases**
    - **Consistency of conceptual schemata (EER, UML etc.)**
    - **Schema integration**
    - **Query subsumption (w.r.t. a conceptual schema)**
  - Ontologies and **Semantic Web** (and **Grid**)
    - **Ontology engineering (design, maintenance, integration)**
    - **Reasoning with ontology-based markup (meta-data)**
    - **Service description and discovery**
- Commercial **implementations**
  - Cerebra system from Network Inference Ltd

From RDF to OWL

- Two languages developed to satisfy the requirements
  - OIL: developed by group of (largely) European researchers (several from EU OntoKnowledge project)
  - DAML-ONT: developed by group of (largely) US researchers (in DARPA DAML programme)
- Efforts merged to produce DAML+OIL
  - Development was carried out by "Joint EU/US Committee on Agent Markup Languages"
  - Extends ("DL subset" of) RDF
- DAML+OIL submitted to W3C as basis for standardisation
  - Web-Ontology (WebOnt) Working Group formed
  - WebOnt group developed OWL language based on DAML+OIL
  - OWL language now a W3C Recommendation (i.e., a standard like HTML and XML)

Description Logic Family

- DLs are a family of logic based KR formalisms
- Particular languages mainly characterised by:
  - Set of constructors for building complex concepts and roles from simpler ones
  - Set of axioms for asserting facts about concepts, roles and individuals
- $\mathcal{ALC}$ is the smallest DL that is propositionally closed
  - Constructors include booleans (and, or, not), and
  - Restrictions on role successors
  - E.g., concept describing "happy fathers" could be written:

$$\textbf{Man} \sqcap \exists \textbf{hasChild.Female} \sqcap \exists \textbf{hasChild.Male} \sqcap \forall \textbf{hasChild.(Rich} \sqcup \textbf{Happy)}$$

DL Concept and Role Constructors

- Range of other constructors found in DLs, including:
  - Number restrictions (cardinality constraints) on roles, e.g., $\geq$3 hasChild, $\leq$1 hasMother
  - Qualified number restrictions, e.g., $\geq$2 hasChild.Female, $\leq$1 hasParent.Male
  - Nominals (singleton concepts), e.g., {Italy}
  - Concrete domains (datatypes), e.g., hasAge.($\leq$ 21)
  - Inverse roles, e.g., hasChild$^-$ (hasParent)
  - Transitive roles, e.g., hasChild* (descendant)
  - Role composition, e.g., hasParent ○ hasBrother (uncle)

DL Knowledge Base

- DL Knowledge Base (KB) normally separated into 2 parts:
  - TBox is a set of axioms describing structure of domain (i.e., a conceptual schema), e.g.:
    - **HappyFather ≡ Man ∧ ∃hasChild.Female ∧ …**
    - **Elephant ≡ Animal ∧ Large ∧ Grey**
    - **transitive(ancestor)**
  - ABox is a set of axioms describing a concrete situation (data), e.g.:
    - **John:HappyFather**
    - **<John,Mary>:hasChild**
- Separation has no logical significance
  - But may be conceptually and implementationally convenient

OWL as DL: Class Constructors

Constructor	DL Syntax	Example	FOL Syntax
intersectionOf	$C_1 \sqcap \ldots \sqcap C_n$	Human ⊓ Male	$C_1(x) \wedge \ldots \wedge C_n(x)$
unionOf	$C_1 \sqcup \ldots \sqcup C_n$	Doctor ⊔ Lawyer	$C_1(x) \vee \ldots \vee C_n(x)$
complementOf	$\neg C$	¬Male	$\neg C(x)$
oneOf	$\{x_1\} \sqcup \ldots \sqcup \{x_n\}$	{john} ⊔ {mary}	$x = x_1 \vee \ldots \vee x = x_n$
allValuesFrom	$\forall P.C$	∀hasChild.Doctor	$\forall y.P(x,y) \rightarrow C(y)$
someValuesFrom	$\exists P.C$	∃hasChild.Lawyer	$\exists y.P(x,y) \wedge C(y)$
maxCardinality	$\leqslant nP$	⩽1hasChild	$\exists^{\leqslant n} y.P(x,y)$
minCardinality	$\geqslant nP$	⩾2hasChild	$\exists^{\geqslant n} y.P(x,y)$

- XMLS datatypes as well as classes in ∀P.C and ∃P.C
  - E.g., ∃hasAge.nonNegativeInteger
- Arbitrarily complex nesting of constructors
  - E.g., Person ⊓ ∀hasChild.Doctor ⊔ ∃hasChild.Doctor

RDFS Syntax

E.g., Person ⊓ ∀hasChild.Doctor ⊔ ∃hasChild.Doctor:

```
<owl:Class>
  <owl:intersectionOf rdf:parseType=" collection">
    <owl:Class rdf:about="#Person"/>
    <owl:Restriction>
      <owl:onProperty rdf:resource="#hasChild"/>
      <owl:toClass>
        <owl:unionOf rdf:parseType=" collection">
          <owl:Class rdf:about="#Doctor"/>
          <owl:Restriction>
            <owl:onProperty rdf:resource="#hasChild"/>
            <owl:hasClass rdf:resource="#Doctor"/>
          </owl:Restriction>
        </owl:unionOf>
      </owl:toClass>
    </owl:Restriction>
  </owl:intersectionOf>
</owl:Class>
```

OWL as DL: Axioms

Axiom	DL Syntax	Example
subClassOf	$C_1 \sqsubseteq C_2$	Human $\sqsubseteq$ Animal $\sqcap$ Biped
equivalentClass	$C_1 \equiv C_2$	Man $\equiv$ Human $\sqcap$ Male
disjointWith	$C_1 \sqsubseteq \neg C_2$	Male $\sqsubseteq \neg$Female
sameIndividualAs	$\{x_1\} \equiv \{x_2\}$	{President_Bush} $\equiv$ {G_W_Bush}
differentFrom	$\{x_1\} \sqsubseteq \neg\{x_2\}$	{john} $\sqsubseteq \neg${peter}
subPropertyOf	$P_1 \sqsubseteq P_2$	hasDaughter $\sqsubseteq$ hasChild
equivalentProperty	$P_1 \equiv P_2$	cost $\equiv$ price
inverseOf	$P_1 \equiv P_2^-$	hasChild $\equiv$ hasParent$^-$
transitiveProperty	$P^+ \sqsubseteq P$	ancestor$^+ \sqsubseteq$ ancestor
functionalProperty	$\top \sqsubseteq \leqslant 1 P$	$\top \sqsubseteq \leqslant$1hasMother
inverseFunctionalProperty	$\top \sqsubseteq \leqslant 1 P^-$	$\top \sqsubseteq \leqslant$1hasSSN$^-$

- Axioms (mostly) reducible to inclusion ($\sqsubseteq$)
  - $C \equiv D$ iff both $C \sqsubseteq D$ and $D \sqsubseteq C$
- Obvious FOL equivalences
  - E.g., $C \equiv D$ iff $\forall x.\ C(x) \Leftrightarrow D(x)$,
  - $C \sqsubseteq D$ iff $\forall x.\ C(x) \Rightarrow D(x)$

XML Schema Datatypes in OWL

- OWL supports XML Schema primitive datatypes
  - E.g., integer, real, string, …
- Strict separation between "object" classes and datatypes
  - Disjoint interpretation domain Δ_D for datatypes
    - For a datavalue d holds $d^\mathcal{I} \subseteq \Delta_D$
    - and $\Delta_D \cap \Delta^\mathcal{I} = \emptyset$
  - Disjoint "object" and datatype properties
    - For a datatype propterty P holds $P^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta_D$
    - For object property S and datatype property P hold $S^\mathcal{I} \cap P^\mathcal{I} = \emptyset$
- Equivalent to the "(D_n)" in $\mathcal{SHOIN}(D_n)$

Why Separate Classes and Datatypes?

- Philosophical reasons:
  - Datatypes structured by built-in predicates
  - Not appropriate to form new datatypes using ontology language
- Practical reasons:
  - Ontology language remains simple and compact
  - Semantic integrity of ontology language not compromised
  - Implementability not compromised — can use hybrid reasoner

OWL DL Semantics

- Mapping OWL to equivalent DL ($\mathcal{SHOIN}(D_n)$):
  - Facilitates provision of reasoning services (using DL systems)
  - Provides well defined semantics
- DL semantics defined by interpretations: $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$, where
  - $\Delta^\mathcal{I}$ is the domain (a non-empty set)
  - $\cdot^\mathcal{I}$ is an interpretation function that maps:
    - **Concept (class) name A to subset $A^\mathcal{I}$ of $\Delta^\mathcal{I}$**
    - **Role (property) name R to binary relation $R^\mathcal{I}$ over $\Delta^\mathcal{I}$**
    - **Individual name i to element $i^\mathcal{I}$ of $\Delta^\mathcal{I}$**

DL Semantics

- Interpretation function $\cdot^{\mathcal{I}}$ extends to concept expressions in the obvious way, i.e.:

$$
\begin{aligned}
(C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} &= C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
(\neg C)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \\
\{x\}^{\mathcal{I}} &= \{x^{\mathcal{I}}\} \\
(\exists R.C)^{\mathcal{I}} &= \{x \mid \exists y.\langle x, y\rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\} \\
(\forall R.C)^{\mathcal{I}} &= \{x \mid \forall y.(x, y) \in R^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\} \\
(\leqslant nR)^{\mathcal{I}} &= \{x \mid \#\{y \mid \langle x, y\rangle \in R^{\mathcal{I}}\} \leqslant n\} \\
(\geqslant nR)^{\mathcal{I}} &= \{x \mid \#\{y \mid \langle x, y\rangle \in R^{\mathcal{I}}\} \geqslant n\}
\end{aligned}
$$

DL Knowledge Bases (Ontologies)

- An OWL ontology maps to a DL Knowledge Base $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$
  - $\mathcal{T}$ (Tbox) is a set of axioms of the form:
    - $C \sqsubseteq D$ (**concept inclusion**)
    - $C \equiv D$ (**concept equivalence**)
    - $R \sqsubseteq S$ (**role inclusion**)
    - $R \equiv S$ (**role equivalence**)
    - $R^{+} \sqsubseteq R$ (**role transitivity**)
  - $\mathcal{A}$ (Abox) is a set of axioms of the form
    - $x \in D$ (**concept instantiation**)
    - $\langle x,y \rangle \in R$ (**role instantiation**)
- Two sorts of Tbox axioms often distinguished
  - "Definitions"
    - **$C \sqsubseteq D$ or $C \equiv D$ where C is a concept name**
  - General Concept Inclusion axioms (GCIs)
    - **$C \sqsubseteq D$ where C in an arbitrary concept**

Knowledge Base Semantics

- An interpretation $\mathcal{I}$ satisfies (models) an axiom A ($\mathcal{I} \vDash A$):
  - $\mathcal{I} \vDash C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
  - $\mathcal{I} \vDash C \equiv D$ iff $C^{\mathcal{I}} = D^{\mathcal{I}}$
  - $\mathcal{I} \vDash R \sqsubseteq S$ iff $R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$
  - $\mathcal{I} \vDash R \equiv S$ iff $R^{\mathcal{I}} = S^{\mathcal{I}}$
  - $\mathcal{I} \vDash R^{+} \sqsubseteq R$ iff $(R^{\mathcal{I}})^{+} \subseteq R^{\mathcal{I}}$
  - $\mathcal{I} \vDash x \in D$ iff $x^{\mathcal{I}} \in D^{\mathcal{I}}$
  - $\mathcal{I} \vDash \langle x,y \rangle \in R$ iff $(x^{\mathcal{I}},y^{\mathcal{I}}) \in R^{\mathcal{I}}$
- $\mathcal{I}$ satisfies a Tbox $\mathcal{T}$ ($\mathcal{I} \vDash \mathcal{T}$) iff $\mathcal{I}$ satisfies every axiom A in $\mathcal{T}$
- $\mathcal{I}$ satisfies an Abox $\mathcal{A}$ ($\mathcal{I} \vDash \mathcal{A}$) iff $\mathcal{I}$ satisfies every axiom A in $\mathcal{A}$
- $\mathcal{I}$ satisfies an KB $\mathcal{K}$ ($\mathcal{I} \vDash \mathcal{K}$) iff $\mathcal{I}$ satisfies both $\mathcal{T}$ and $\mathcal{A}$

Inference Tasks

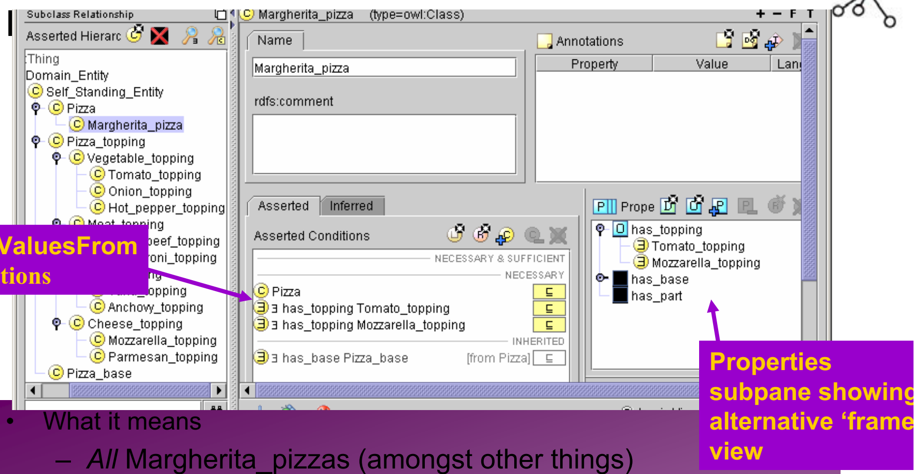
- Knowledge is correct (captures intuitions)
  - C subsumes D w.r.t. $\mathcal{K}$ iff for *every* model $\mathcal{I}$ of $\mathcal{K}$, $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
- Knowledge is minimally redundant (no unintended synonyms)
  - C is equivalent to D w.r.t. $\mathcal{K}$ iff for *every* model $\mathcal{I}$ of $\mathcal{K}$, $C^{\mathcal{I}} = D^{\mathcal{I}}$
- Knowledge is meaningful (classes can have instances)
  - C is satisfiable w.r.t. $\mathcal{K}$ iff there exists *some* model $\mathcal{I}$ of $\mathcal{K}$ s.t. $C^{\mathcal{I}} \neq \emptyset$
- Querying knowledge
  - x is an instance of C w.r.t. $\mathcal{K}$ iff for *every* model $\mathcal{I}$ of $\mathcal{K}$, $x^{\mathcal{I}} \in C^{\mathcal{I}}$
  - $\langle x,y \rangle$ is an instance of R w.r.t. $\mathcal{K}$ iff for, *every* model $\mathcal{I}$ of $\mathcal{K}$, $(x^{\mathcal{I}},y^{\mathcal{I}}) \in R^{\mathcal{I}}$
- Knowledge base consistency
  - A KB $\mathcal{K}$ is consistent iff there exists *some* model $\mathcal{I}$ of $\mathcal{K}$

DL Reasoning

- Tableau algorithms used to test satisfiability (consistency)
- Try to build a tree-like model I of the input concept C
- Decompose C syntactically
  - Apply tableau expansion rules
  - Infer constraints on elements of model
- Tableau rules correspond to constructors in logic ($\sqcap$, $\sqcup$ etc)
  - Some rules are nondeterministic (e.g., $\sqcup$, $\leqslant$)
  - In practice, this means search
- Stop when no more rules applicable or clash occurs
  - Clash is an obvious contradiction, e.g., $A(x)$, $\neg A(x)$
- Cycle check (blocking) may be needed for termination
- C satisfiable iff rules can be applied such that a fully expanded clash free tree is constructed

Highly Optimised Implementation

- Naive implementation leads to effective non-termination
- Modern systems include MANY optimisations
- Optimised classification (compute partial ordering)
  - Use enhanced traversal (exploit information from previous tests)
  - Use structural information to select classification order
- Optimised subsumption testing (search for models)
  - Normalisation and simplification of concepts
  - Absorption (rewriting) of general axioms
  - Davis-Putnam style semantic branching search
  - Dependency directed backtracking
  - Caching of satisfiability results and (partial) models
  - Heuristic ordering of propositional and modal expansion
  - …



- What it means
  - *All* Margherita_pizzas (amongst other things)
    - ***Are* Pizzas**
    - **have_topping *some* Tomato_topping**
    - **have_topping *some* Mozzarella_topping**
      - & because they are Pizzas have_base *some* Pizza_base