Levelwise Search of Frequent Patterns with Counting Inference

Yves Bastide,[1] Rafik Taouil,[1] Nicolas Pasquier,[1]
Gerd Stumme,[2] Lotfi Lakhal[1]

[1] Laboratoire LIMOS, FRE CNRS 2239, université Blaise Pascal,
complexe scientifique des Cézeaux, 24 av. des Landais,
63 177 Aubière Cedex, France;
{bastide,taouil,pasquier,lakhal}@libd2.univ-bpclermont.fr
[2] Technische Universität Darmstadt, Fachbereich Mathematik,
Schloßgartenstr. 7, D–64289 Darmstadt, Germany;
stumme@mathematik.tu-darmstadt.de

Abstract

In this paper, we propose the algorithm Pascal which introduces a novel optimization of the well-known algorithm Apriori. Being provided with a given *minsup* threshold, Pascal discovers all frequent patterns by performing as few counting as possible. In order to derive the support of larger patterns *without accessing the database* whenever it is possible, we use the knowledge about the support of some of their sub-patterns, the so-called *key patterns*. Experiments comparing Pascal to the three algorithms Apriori, Close and Max-Miner, each of which being representative of a frequent patterns discovery strategy, show that Pascal is the most efficient algorithm for extracting patterns from strongly correlated data. Moreover, its execution times are equivalent to those of Apriori and Max-Miner when data is weakly correlated.

Keywords: Data mining, database theory, algorithms, performance analysis.

1 Introduction

Knowledge discovery in databases (KDD) is defined as the non-trivial extraction of valid, implicit, potentially useful and ultimately understandable information in large databases. For several years, a wide range of applications in various domains have benefited from KDD techniques and many work has been conducted on this topic. The problem of mining frequent patterns arose first as a sub-problem of mining association rules, but then it turned out that frequent patterns solve a variety of problems: mining sequential patterns [AS95], episodes [MTV97], association rules [AS94], correlations [BMS97, SBM98], multi-dimensional patterns [KHC97, LSW97], maximal patterns [ZPOL97, LK98] and several other important knowledge discovery tasks [HPY00]. Since the complexity of this problem is exponential in the size of the binary database input relation and since this relation has to be scanned several times during the process, efficient algorithms for mining frequent patterns are required.

1.1 Related work

Three approaches have been proposed for mining frequent patterns: The first is traversing iteratively the set of all patterns in a levelwise manner. During each iteration corresponding to a level, a set of candidate patterns is created by joining the frequent patterns discovered during the previous iteration, the supports of all candidate patterns are counted and infrequent ones are discarded. The most prominent algorithm based on this approach is the Apriori algorithm [AS94], that is identical to the algorithm OCD [MTV94] proposed concurrently. A variety of modifications of this algorithm arose [BMUT97, GPW98, PCY95, SON95, Toi96] in order to improve different efficiency aspects. However, all of these algorithms have to determine the supports of *all* frequent patterns and of some infrequent ones in the database.

The second approach is based on the extraction of maximal[1] frequent patterns, from which all supersets are infrequent and all subsets are frequent. This approach combines a levelwise bottom-up traversal with a top-down traversal in order to quickly find the maximal frequent patterns. Then, all frequent patterns are derived from these ones and one last database scan is carried on to count their support. The most prominent algorithm using this approach is Max-Miner [Bay98]. Experimental results have shown that this approach is particularly efficient for extracting maximal frequent patterns, but when applied for extracting all frequent patterns performances drastically decrease because of the cost of the last scan which requires roughly an inclusion test between each frequent pattern and each object of the database. As algorithms based on the first approach, algorithms based on this approach have to extract the supports of all frequent patterns from the database.

The third approach, represented by the Close algorithm [PBTL99], is based on the theoretical framework introduced in [PBTL98] that uses the closure of the Galois connection [GW99]. In this approach, the frequent closed patterns (and their support) are extracted from the database in a levelwise manner. A closed pattern is a pattern that is common to a set of objects of the database and each non-closed pattern has the same properties (same set of objects containing it and thus same support) as the smallest closed pattern containing it that is its closure. Then, all frequent patterns as well as their support are derived from the frequent closed patterns and their support *without accessing the database*. Hence not all patterns are considered during the most expensive part of the algorithm (counting the supports of the patterns) and the search space is drastically reduced, especially for strongly correlated data. Experiments have shown that this approach is much more efficient than the two previous ones on such data.

1.2 Contribution

In this paper, we present the PASCAL [2] algorithm, introducing a novel, effective and simple optimization of the algorithm Apriori. This optimization is based on *pattern counting inference* that relies on the concept of *key patterns*. A key pattern is a minimal pattern of an equivalence class gathering all patterns that have the

[1] 'Maximal' means 'maximal with respect to set inclusion'.

[2] The French mathematician Blaise Pascal (* Clermont-Ferrand 1623, † 1662 Paris) invented an early computing device.

same objects. The pattern counting inference allows to determine the supports of *some* frequent and infrequent patterns (the key patterns) in the database only. The supports of all other frequent patterns are derived from the frequent key patterns. This allows to reduce, at each database pass, the number of patterns considered, and, even more important, to reduce the number of passes in total. This optimization is valid since key patterns have a property that is compatible with the pruning of Apriori: all subsets of a key pattern are key patterns and all supersets of a non-key pattern are non-key patterns. Then, the counting inference is performed in a levelwise manner: If a candidate pattern of size k which support has to be determined is a non-key pattern, then its support is equal to the minimal support among the patterns of size k-1 that are its subsets. In comparison to most other modifications of Apriori, this results in a minimal impact on the understandability and simplicity of implementation of the algorithm. The important difference is to determine as much support counts as possible without accessing the database by information gathered in previous passes. As shown by the experiments, the efficiency gain is up to the order of a magnitude on correlated data.

1.3 Organization of the paper

In the next section, we recall the problem of mining frequent patterns. The essential notions and the definitions of key patterns and pattern counting inference are given in Section 3. The Pascal algorithm is described in Section 4 and experimental results for comparing its efficiency to those of Apriori, Max-Miner and Close are presented in Section 5. A summary of the paper and some perspectives of future work are given in Section 6.

2 Recall: The Problem of Mining Frequent Patterns

Definition 1. Let $\mathbb{P}$ be a finite set of *items*, $\mathbb{O}$ a finite set of *objects* (e. g., transaction ids) and $\mathbb{R} \subseteq \mathbb{O}\times\mathbb{P}$ a binary relation between both (where $(o,p) \in \mathbb{R}$ may for instance be read as "item p is included in transaction o"). The triple $\mathbb{D} = (\mathbb{O},\mathbb{P},\mathbb{R})$ is called *dataset*.

Each subset P of $\mathbb{P}$ is called a *pattern*. We say that a pattern P is *included* in an object $o \in \mathbb{O}$ if $(o,p) \in \mathbb{R}$ for all $p \in P$. Let f be the function which assigns to each pattern $P \subseteq \mathbb{P}$ the set of all objects which include this pattern: $f(P) = \{o \in \mathbb{O} \mid o \text{ includes } P\}$.

The *support* of a pattern P is given by $\text{supp}(P) = \frac{\text{card}(f(P))}{\text{card}(\mathbb{O})}$. For a given threshold $\text{minsup} \in [0,1]$, a pattern P is called *frequent pattern* if $\text{supp}(P) \geq \text{minsup}$.

Problem: The task of mining frequent patterns consists in determining all frequent patterns together with their supports[3] for a given threshold minsup.

[3] There are also application where the supports need not be known exactly. We only consider the case where all supports have to be determined as well.

3 Key Patterns and Pattern Counting Inference

In this section, we give the theoretical basis of the new PASCAL algorithm. This basis provides at the same time the proof of correctness of the algorithm. The following theorems are turned into pseudo-code in the Section 4.

As Apriori, PASCAL will traverse the powerset of $\mathbb{P}$ levelwise: At the k^{th} iteration, the algorithm generates first all *candidate k-patterns.*

Definition 2. A k-pattern P is a subset P of $\mathbb{P}$ with $\text{card}(P) = k$. A *candidate k-pattern* is a k-pattern where all its proper sub-patterns are frequent.

For the candidate k-patterns one database pass is used to determine their support. Then infrequent patterns are pruned. This approach works because the well-known fact that a pattern cannot be frequent if it has an infrequent sub-pattern.

3.1 Key Patterns

Our approach is based on the observation that frequent patterns can be considered as "equivalent" if they are included in exactly the same objects. We describe this fact by the following equivalence relation θ on the frequent patterns.

Definition 3. For frequent patterns $P, Q \subseteq \mathbb{P}$, we let $P\,\theta\,Q$ if $f(P) = f(Q)$. The set of patterns which are *equivalent* to a pattern P is given by $[P] = \{Q \subseteq \mathbb{P} \mid P\,\theta\,Q\}$.

In the case of frequent patterns P and Q with $P\,\theta\,Q$, both patterns have obviously the same support:

Lemma 1. *Let P and Q be frequent patterns.*
(i) $\quad P\,\theta\,Q \implies \text{supp}(P) = \text{supp}(Q)$
(ii) $\quad P \subseteq Q \wedge \text{supp}(P) = \text{supp}(Q) \implies P\,\theta\,Q$

Proof. (i) $\quad P\,\theta\,Q \iff f(P) = f(Q) \implies \text{supp}(P) = \frac{\text{card}(f(P))}{\text{card}(\mathbb{O})} = \frac{\text{card}(f(Q))}{\text{card}(\mathbb{O})} = \text{supp}(Q)$.

(ii) Since $P \subseteq Q$ and f is monotonous decreasing, we have $f(P) \supseteq f(Q)$. $\text{supp}(P) = \text{supp}(Q)$ is equivalent to $\text{card}(f(P)) = \text{card}(f(Q))$ which implies with the former $f(P) = f(Q)$ and thus $P\,\theta\,Q$. $\square$

Hence if we knew the relation θ in advance, we would need to count the support of only one pattern in each equivalence class. Of course we do not know the relation in advance, but we can construct it step by step.[4] Thus, we will in general need to determine the support of more than one pattern in each class, but not of all of them. If we already have determined the support of a pattern P in the database and pass later a pattern $Q \in [P]$, then we need not access the database for it because we know that $\text{supp}(Q) = \text{supp}(P)$.

The first patterns of an equivalence class that we reach using a levelwise approach are exactly the minimal[5] patterns in the class:

[4] In the algorithm, the equivalence relation is not explicitly generated, but is – as the algorithm is based on the following theorems – implicitly used.

[5] 'Minimal' means 'minimal with respect to set inclusion'.

Definition 4. A frequent pattern P is a *key pattern* if $P \in \min[P]$. A *candidate key pattern* is a pattern where all its proper sub-patterns are key patterns.

Observe that all candidate key patterns are obviously also candidate patterns.

3.2 Pattern Counting Inference

In the algorithm we apply the pruning strategy both for candidate patterns and to candidate key patterns. This is justified by the following theorem.

Theorem 2. (i) *If Q is a key pattern and $P \subseteq Q$, then P is also a key pattern.* (ii) *If P is not a key pattern and $P \subseteq Q$, then Q is not a key pattern either.*[6]

Proof. (ii) Let $P \subseteq Q$ and P be not a key pattern. Then exists $P' \in \min[P]$ with $P' \subset P$. From $f(P') = f(P)$ it follows $f(Q) = f(Q \setminus (P \setminus P'))$. Hence Q is not minimal in $[Q]$ and thus by definition not a key pattern. (i) is a direct logical consequence of (ii). □

The algorithm determines, at each iteration, the key patterns among the candidate key patterns by using (ii) of the following theorem:

Theorem 3. *Let P be a frequent pattern.*
(i) *Let $p \in P$. Then $P \in [P \setminus \{p\}]$ if and only if* $\mathrm{supp}(P) = \mathrm{supp}(P \setminus \{p\})$.
(ii) *P is a key pattern if and only if* $\mathrm{supp}(P) \neq \min_{p \in P}(\mathrm{supp}(P \setminus \{p\}))$.

Proof. (i) The "if" part follows from Lemma 1 (ii). The "only if" part is obvious. (ii) From (i) we deduce that P is a key pattern if and only if $\mathrm{supp}(P) \neq \mathrm{supp}(P \setminus \{p\})$, for all $p \in P$. Since supp is a monotonous decreasing function, this is equivalent to (ii). □

Since all candidate key patterns are also candidate patterns, when generating all candidate patterns for the next level we can at the same time determine the candidate key patterns among them.

If we reach a candidate k-pattern which is not a candidate key pattern, then we already passed along at least one of the key patterns in its equivalence class in an earlier iteration. Hence we already know its support. Using the following theorem, we determine this support *without accessing the database*:

Theorem 4. *If P is a non-key pattern, then*

$$\mathrm{supp}(P) = \min_{p \in P}(\mathrm{supp}(P \setminus \{p\})) \ .$$

Proof. "$\leq$" follows from the fact that supp is a monotonous decreasing function. "$\geq$": If P is not a key pattern then exists $p \in P$ with $P \,\theta\, P \setminus \{p\}$. Hence $\mathrm{supp}(P) = \mathrm{supp}(P \setminus \{p\}) \geq \min_{q \in P}(\mathrm{supp}(P \setminus \{q\})$. □

[6] In mathematical terms, (i) and (ii) state that the set of key patterns is an order ideal (or down-set) of $(2^{\mathbb{P}}, \subseteq)$.

Thus the database pass needs to count the supports of the candidate key patterns only. Knowing this, we can summarize PASCAL as follows: It works exactly as Apriori, but counts only those supports in the database pass which cannot be derived from supports already computed. Thus we can, on each level, restrict the expensive count in the database to some of the candidates. But even better, from some level on, all candidate pattern may be known to be non-key patterns. Then all remaining frequent patterns and their support can be derived without accessing the database any more. In the worst case (i. e., in weakly correlated data), all candidates patterns are also candidate key patterns. Then the algorithm behaves exactly as Apriori without any overhead.

4 The PASCAL algorithm

In this section, we transform the theorems given in the last section into an algorithm. The pseudo-code is given in Algorithm 1. A list of notations is provided in Table 1. We assume that $\mathbb{P}$ is linearly ordered, e. g., $\mathbb{P} = \{1, \ldots, n\}$. This will be used in PASCAL-GEN.

Table 1: Notations used in PASCAL

k	is the counter which indicates the current iteration. In the kth iteration, all frequent k-patterns and all key patterns among them are determined.
$\mathcal{P}_k$	contains after the kth iteration all frequent k-patterns P together with their support P.supp, and a boolean variable P.key indicating if P is a (candidate) key pattern.
$\mathcal{C}_k$	stores the candidate k-patterns together with their support (if known), the boolean variable P.key, and a counter P.pred_supp which stores the minimum of the supports of all $(k-1)$-sub-patterns of P.

The algorithm starts with the empty set, which always has a support of 1 and which is (by definition) a key pattern (steps 1+2). In step 3, frequent 1-patterns are determined. They are marked as key patterns unless their support is 1 (steps 4–6). The main loop is similar to the one in Apriori (steps 7 to 21). First, PASCAL-GEN is called to compute the candidate patterns. The support of key ones is determined via a database pass (steps 10–14).

Then (steps 15–20) the 'traditional' pruning (step 16) is done. At the same time, for all remaining candidate key patterns, it is determined whether they are key or not (steps 17+18).

The way that PASCAL-GEN operates is basically known from the generator function Apriori-Gen which was introduced in [AS94]. When called at the kth iteration, it uses as input the set of frequent $(k-1)$-patterns $\mathcal{P}_{k-1}$. Its output is the set of candidate k-patterns. Additionally to Apriori-Gen's *join* and *prune* steps, PASCAL-GEN makes the new candidates inherit the fact of being or not a candidate key pattern (step 9) by using Theorem 2; and it determines at the same time the support of all non key candidate patterns (step 12) by using Theorem 4.

Algorithm 1 PASCAL

$\emptyset$.supp $\leftarrow 1$; $\emptyset$.key $\leftarrow$ true;
$\mathcal{P}_0 \leftarrow \{\emptyset\}$;
$\mathcal{P}_1 \leftarrow \{$frequent 1-patterns$\}$;
forall $p \in \mathcal{P}_1$ **do begin**
  p.pred_supp $\leftarrow 1$; p.key $\leftarrow$ (p.supp $\neq 1$);
end;
for $(k = 2;\ \mathcal{P}_{k-1} \neq \emptyset;\ k++)$ **do begin**
  $\mathcal{C}_k \leftarrow$ PASCAL-GEN($\mathcal{P}_{k-1}$);
  if $\exists c \in \mathcal{C}_k \mid c$.key **then**
    forall $o \in \mathbb{D}$ **do begin**
      $\mathcal{C}_o \leftarrow$ subset($\mathcal{C}_k, o$);
      forall $c \in \mathcal{C}_p \mid c$.key **do**
        c.supp $++$;
    end;
  forall $c \in \mathcal{C}_k$ **do**
    if c.supp $\geq$ minsup **then begin**
      if c.key and c.supp $= c$.pred_supp **then**
        c.key $\leftarrow$ false;
      $\mathcal{P}_k \leftarrow \mathcal{P}_k \cup \{c\}$;
    end;
end;
return $\bigcup_k \mathcal{P}_k$.

Running example. We illustrate the PASCAL algorithm on the following dataset for minsup $= 2/5$:

ID	Items				
1	A	C	D	F	
2	B	C	E	F	
3	A	B	C	E	F
4	B	E	F		
5	A	B	C	E	F

The algorithm performs first one database pass to count the support of the 1-patterns. The candidate pattern $\{D\}$ is pruned because it is infrequent. As $\{F\}$ has the same support as the empty set, $\{F\}$ is marked as a non-key pattern:

$\mathcal{P}_1$	supp	key
$\{A\}$	3/5	t
$\{B\}$	4/5	t
$\{C\}$	4/5	t
$\{E\}$	4/5	t
$\{F\}$	1	f

At the next iteration, all candidate 2-patterns are created and stored in $\mathcal{C}_2$. At the same time, the support of all patterns containing $\{F\}$ as sub-pattern is

Algorithm 2 PASCAL-GEN

Input: $\mathcal{P}_{k-1}$, the set of frequent $(k-1)$-patterns p with their support p.supp and the p.key flag.

Output: $\mathcal{C}_k$, the set of candidate k-patterns c each with the flag c.key, the value c.pred_supp, and the support c.supp if c is not a key pattern.

1) **insert into** $\mathcal{C}_k$
   select $p.\text{item}_1, p.\text{item}_2, \ldots, p.\text{item}_{k-1}, q.\text{item}_{k-1}$
   from $\mathcal{P}_{k-1}\ p, \mathcal{P}_{k-1}\ q$
   where $p.\text{item}_1 = q.\text{item}_1, \ldots, p.\text{item}_{k-2} = q.\text{item}_{k-2}, p.\text{item}_{k-1} < q.\text{item}_{k-1}$;
2) **forall** $c \in \mathcal{C}_k$ **do begin**
3) $\quad c$.key $\leftarrow$ true; c.pred_supp $\leftarrow +\infty$;
4) $\quad$ **forall** $(k-1)$-subsets s of c **do begin**
5) $\quad\quad$ **if** $s \notin \mathcal{P}_{k-1}$ **then**
6) $\quad\quad\quad$ **delete** c from $\mathcal{C}_k$;
7) $\quad\quad$ **else begin**
8) $\quad\quad\quad c$.pred_supp $\leftarrow \min(c.\text{pred\_supp}, s.\text{supp})$;
9) $\quad\quad\quad$ **if** not s.key **then** c.key $\leftarrow$ false;
10) $\quad\quad$ **end**;
11) $\quad$ **end**;
12) $\quad$ **if** not c.key **then** c.supp $\leftarrow$ c.pred_supp;
13) **end**;
14) **return** $\mathcal{C}_k$.

computed. Then a database pass is performed to determine the supports of the remaining six candidate patterns:

$\mathcal{C}_2$	pred_supp	key	supp
$\{AB\}$	3/5	t	?
$\{AC\}$	3/5	t	?
$\{AE\}$	4/5	t	?
$\{AF\}$	3/5	f	3/5
$\{BC\}$	4/5	t	?
$\{BE\}$	4/5	t	?
$\{BF\}$	4/5	f	4/5
$\{CE\}$	4/5	t	?
$\{CF\}$	4/5	f	4/5
$\{EF\}$	4/5	f	4/5

$\mathcal{P}_2$	supp	key
$\{AB\}$	2/5	t
$\{AC\}$	3/5	f
$\{AE\}$	2/5	t
$\{AF\}$	3/5	f
$\{BC\}$	3/5	t
$\{BE\}$	4/5	f
$\{BF\}$	4/5	f
$\{CE\}$	3/5	t
$\{CF\}$	4/5	f
$\{EF\}$	4/5	f

At the third iteration, it turns out in PASCAL-GEN that each newly generated candidate pattern contains at least one sub-pattern which is not a key pattern. Hence all new candidate patterns are no candidate key patterns. All their supports are determined directly in PASCAL-GEN. *From that moment on, the database will not be accessed any more.*

$\mathcal{C}_3$	pred_supp	key	supp
$\{ABF\}$	2/5	f	2/5
$\{ABC\}$	2/5	f	2/5
$\{ABE\}$	2/5	f	2/5
$\{ACE\}$	2/5	f	3/5
$\{ACF\}$	3/5	f	3/5
$\{AEF\}$	2/5	f	2/5
$\{BCE\}$	3/5	f	3/5
$\{BCF\}$	3/5	f	3/5
$\{BEF\}$	4/5	f	4/5
$\{CEF\}$	3/5	f	3/5

$\mathcal{P}_3$	supp	key
$\{ABF\}$	2/5	f
$\{ABC\}$	2/5	f
$\{ABE\}$	2/5	f
$\{ACE\}$	2/5	f
$\{ACF\}$	3/5	f
$\{AEF\}$	2/5	f
$\{BCE\}$	3/5	f
$\{BCF\}$	3/5	f
$\{BEF\}$	4/5	f
$\{CEF\}$	3/5	f

In the fourth and fifth iteration, all supports are determined directly in PASCAL-GEN. In the sixth iteration, PASCAL-GEN generates no new candidate patterns, thus no frequent 6-patterns are computed and the algorithm stops:

$\mathcal{C}_4$	pred_supp	key	supp
$\{ABCE\}$	2/5	f	2/5
$\{ABCF\}$	2/5	f	2/5
$\{ABEF\}$	2/5	f	2/5
$\{ACEF\}$	2/5	f	3/5
$\{BCEF\}$	3/5	f	3/5

$\mathcal{P}_4$	supp	key
$\{ABCE\}$	2/5	f
$\{ABCF\}$	2/5	f
$\{ABEF\}$	2/5	f
$\{ACEF\}$	2/5	f
$\{BCEF\}$	3/5	f

$\mathcal{C}_5$	pred_supp	key	supp
$\{ABCEF\}$	2/5	f	2/5

$\mathcal{P}_5$	supp	key
$\{ABCEF\}$	2/5	f

Hence PASCAL needs two database passes in which the algorithm counted the supports of $6+6=12$ patterns. Apriori would have needed five database passes for counting the supports of $6+10+10+5+1=32$ patterns for the same dataset. All other current algorithms (with the only exception of Close) may need less then five passes, but they all have to perform the 32 counts.

5 Experimental Evaluation

We evaluated PASCAL against three algorithms, each representative of one frequent patterns discovery strategy: Apriori, Close, and Max-Miner. This Max-Miner implementation was kindly provided by Roberto Bayardo, and retrieving the frequent patterns' support from the maximal frequent ones was done using a brute-force method[7]. PASCAL, Apriori, Close and this final step to Max-Miner all shared the same data structures and general organization. Optimizations such as special handling of pass two were disabled.

Characteristics of the datasets used are given in Table 2. These datasets are the C20D10K and C73D10K census datasets from the PUMS sample file[8], the T20I6D100K, T25I10D10K and T25I20D100K[9] synthetic dataset that mimics mar-

[7] In the following tables, we distinguished the time spent by Max-Miner itself and the support retrieval step.

[8] `ftp://ftp2.cc.ukans.edu/pub/ippbr/census/pums/pums90ks.zip`

[9] `http://www.almaden.ibm.com/cs/quest/syndata.html`

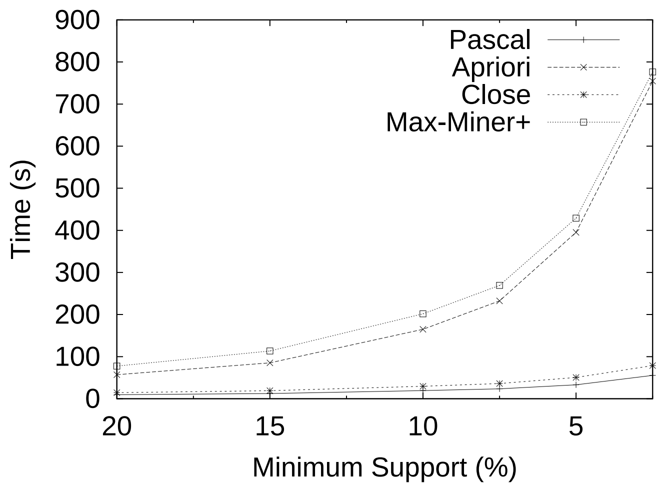
ket basket data, and the MUSHROOMS[10] dataset describing mushrooms characteristics [UCI99]. In all experiments, we attempted to choose significant minimum support threshold values: we observed threshold values used in other papers for experiments on similar data types.

Name	Number of objects	Average size of objects	Number of items
C20D10K	10,000	20	386
C73D10K	10,000	73	2,178
MUSHROOMS	8,416	23	128
T20I6D100K	100,000	20	1,000
T25I10D10K	10,000	25	1,000
T25I20D100K	100,000	25	10,000

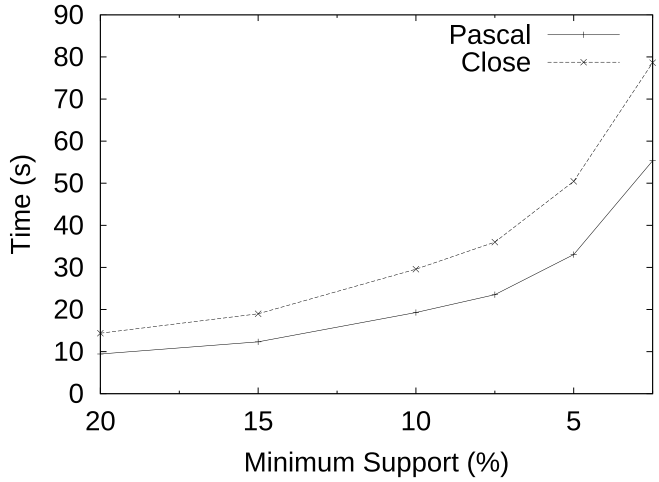
Table 2: Datasets.

C20D10K

Support	# frequents	Pascal	Apriori	Close	Max-Miner	
20.0	20,239	9.44	57.15	14.36	0.17	77.40
15.0	36,359	12.31	85.35	18.99	0.26	113.22
10.0	89,883	19.29	164.81	29.58	0.34	201.33
7.5	153,163	23.53	232.40	36.02	0.35	268.80
5.0	352,611	33.06	395.32	50.46	0.48	428.65
2.5	1,160,363	55.33	754.64	78.63	0.81	775.56

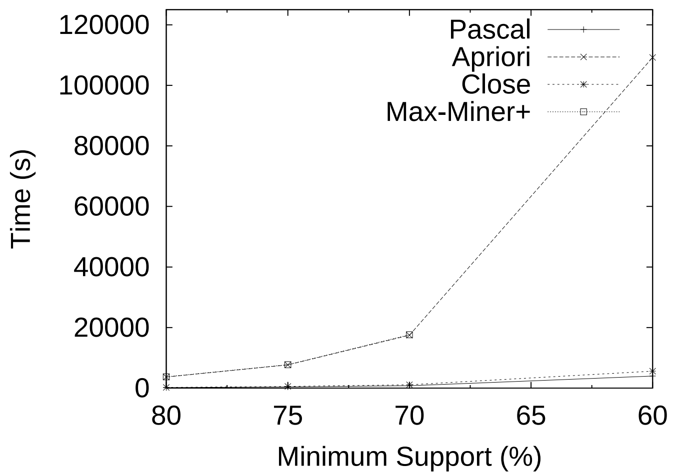


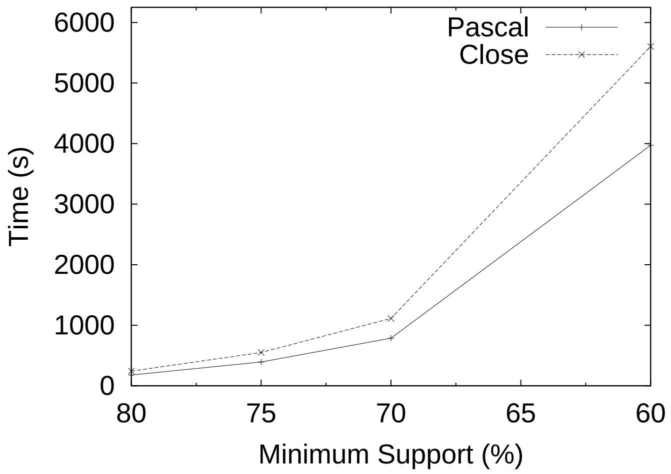
[10] ftp://ftp.ics.uci.edu/pub/machine-learning-databases/mushroom/agaricus-lepiota.data



C73D10K

Support	# frequents	Pascal	Apriori	Close	Max-Miner	
80	109,159	177.49	3,661.27	241.91	0.87	3,717.99
75	235,271	392.80	7,653.58	549.27	1.06	7,730.36
70	572,087	786.49	17,465.10	1,112.42	2.28	17,618.40
60	4,355,543	3,972.10	109,204.00	5,604.91	7.72	

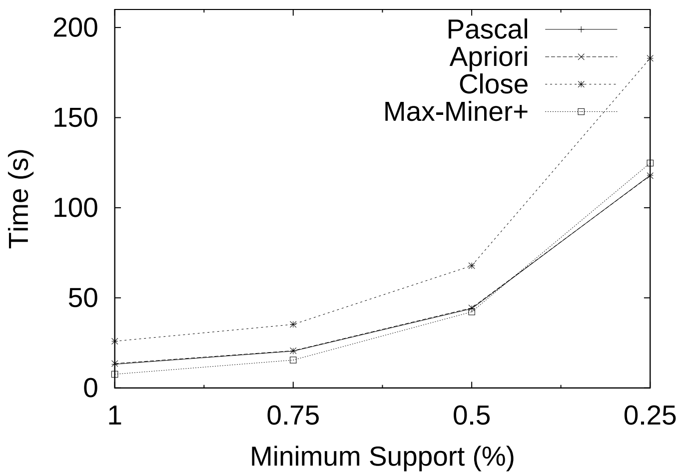




On these two databases, Pascal and Close outperform Apriori and Max-Miner by a wide margin. On C73D10K with minsup = 60%, for instance, they both make 13 passes while the largest frequent patterns are of size 19.

T20I6D100K

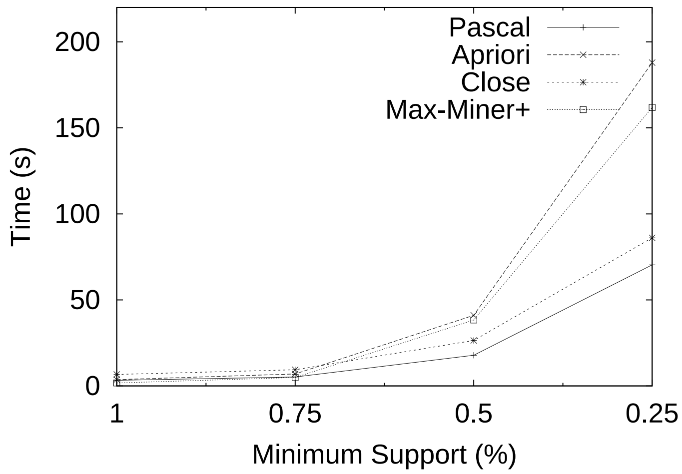
Support	# frequents	Pascal	Apriori	Close	Max-Miner	
1.00	1,534	13.14	13.51	25.91	2.60	5.03
0.75	4,710	20.41	20.67	35.29	4.44	11.06
0.50	26,950	44.00	44.38	67.82	6.87	35.37
0.25	155,673	117.97	117.79	182.95	15.64	109.14



This T20I6D100K database is a typical case where all frequents patterns are key. Here, Pascal, Apriori and Max-Miner are on a par, while Close spends much time computing intersections.

T25I10D10K

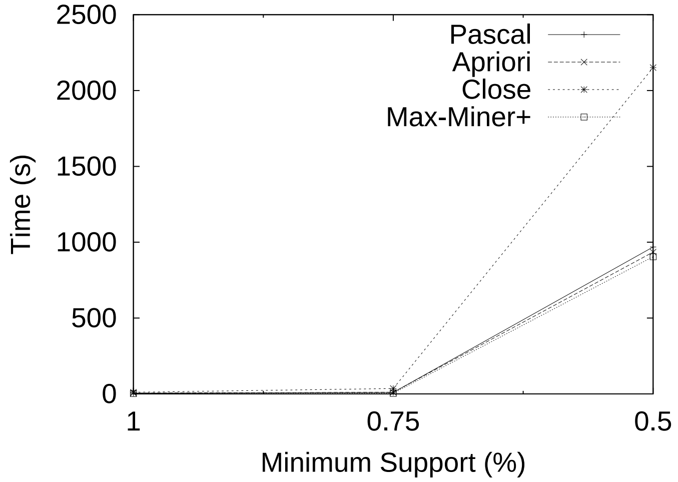
Support	# frequents	Pascal	Apriori	Close	Max-Miner	
1.00	3,300	3.24	3.62	6.67	0.63	1.05
0.75	17,583	5.17	6.95	9.38	1.09	3.83
0.50	331,280	17.82	41.06	26.43	2.76	35.53
0.25	2,270,573	70.37	187.92	86.08	6.99	154.89



T25I10D10K is a basket market database with lots of non key patterns: hence, Pascal is faster than Close, Apriori and Max-Miner.

T25I20D100K

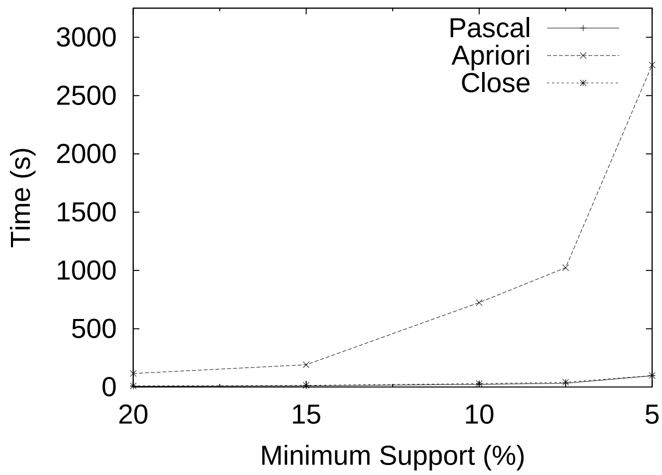
Support	# frequents	Pascal	Apriori	Close	Max-Miner	
1.00	583	5.15	5.76	11.15	1.24	1.3
0.75	1,155	9.73	11.13	35.67	1.99	1.77
0.50	1,279,254	968.64	935.14	2,151.34	24.94	879.85

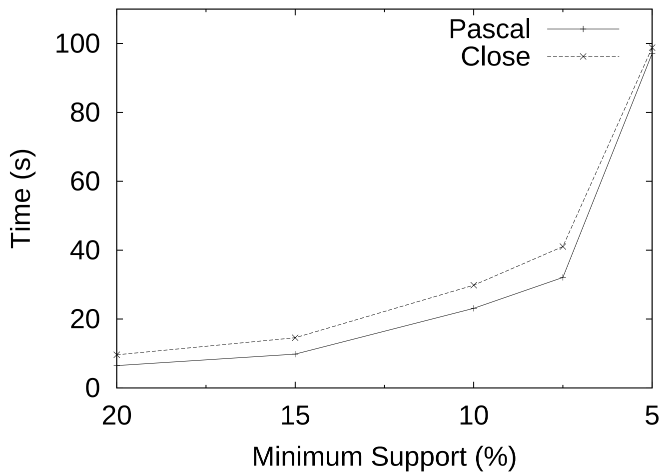


T25I20D100K is like T20I6D100K: Nearly all frequent patterns are key, thus Pascal suffers a slight performance loss over Max-Miner and Apriori while Close is by far the worst performer.

Mushrooms

Support	# frequents	Pascal	Apriori	Close	Max-Miner	
20.0	53,337	6.48	115.82	9.63	0.31	134.31
15.0	99,079	9.81	190.94	14.57	0.50	218.93
10.0	600,817	23.12	724.35	29.83	0.89	745.72
7.5	936,247	32.08	1,023.24	41.05	1.25	1,035.48
5.0	4,140,453	97.12	2,763.42	98.81	1.99	2,752.05





On the Mushrooms database, as the census databases, Pascal and Close are an order of magnitude faster than the two other algorithms.

6 Conclusion

We presented a new algorithm, called Pascal, for efficiently extracting frequent patterns in large databases. This algorithm is a novel, effective and simple optimization of the Apriori algorithm, thus easy to implement or to integrate in an existing implementation based on the Apriori approach. This optimization is based on the notion of key patterns of equivalence classes of patterns. Using these key patterns we propose a method, called pattern counting inference, that allows to determine the support of some frequent patterns, the frequent key patterns, rather than counting the support of all frequent patterns as in algorithms based on the levelwise extraction of frequent patterns or on the extraction of maximal frequent patterns.

We conducted performance evaluations to compare the efficiency of Pascal with those of optimized versions of Apriori, Max-Miner and Close, each one representative of an approach for extracting frequent patterns. The results showed that Pascal gives response times equivalent to those of Apriori and Max-Miner when extracting all frequent patterns and their support from weakly correlated, and that it is the most efficient among the four algorithms when data are correlated.

We think that an important perspective of future work is the integration of pattern counting inference in Database Management Systems. The integration of data mining methods in relational and object database systems is an important research topic [STA98]. Implementing the Pascal algorithm in SQL or OQL, we can benefit from database indexing and query processing capabilities, parallelization of the process (e. g., in a SMP environment) and using support for checkpointing and space management offered by DBMS for instance.

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