Vorlesung Künstliche Intelligenz Wintersemester 2008/09

Teil III: Wissensrepräsentation und Inferenz

Kap.10: Beschreibungslogiken

Mit Material von
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Beschreibungslogiken (Description Logics)

Beschreibungslogiken

- sind eine Familie von logik-basierten Wissensrepräsentationssprachen
- stammen von semantischen Netzen und KL-ONE ab.
- beschreiben die Welt mit Konzepten (Klassen), Rollen (Relationen) und Individuen.

- haben eine formale (typischerweise modell-theoretische) Semantik.
  - Sie sind entscheidbare Fragmente der PL1
  - und eng verwandt mit aussagenlogischen Modal- und Temporallogiken.
- bieten Inferenzmechanismen für zentrale Probleme.
  - Korrekte und vollständige Entscheidungsverfahren existieren.
  - Hoch-effiziente Implementierungen existieren.

- Einfache Sprache zum Start: $\mathcal{ALC}$ (Attributive Language with Complement)
- Im Semantic Web wird $\mathcal{SHOIN}(\mathbf{D}_n)$ eingesetzt. Hierauf basiert die Semantik von OWL DL.

Geschichte

- Ihre Entwicklung wurde inspiriert durch semantische Netze und Frames.
- Frühere Namen:
  - KL-ONE like languages
  - terminological logics

- Ziel war eine Wissensrepräsentation mit formaler Semantik.

- Das erste Beschreibungslogik-basierte System war KL-ONE (1985).
- Weitere Systeme u.a. LOOM (1987), BACK (1988), KRIS (1991), CLASSIC (1991), FaCT (1998), RACER (2001), KAON 2 (2005).

Literatur

- D. Nardi, R. J. Brachman. An Introduction to Description Logics. In: F. Baader, D. Calvanese, D.L. McGuinness, D. Nardi, P.F. Patel-Schneider (eds.): Description Logic Handbook, Cambridge University Press, 2002, 5-44.
- F. Baader, W. Nutt: Basic Description Logics. In: Description Logic Handbook, 47-100.
- Ian Horrocks, Peter F. Patel-Schneider and Frank van Harmelen. From SHIQ and RDF to OWL: The making of a web ontology language. http://www.cs.man.ac.uk/%7Ehorrocks/Publications/download/2003/HoPH03a.pdf

What is the Problem?

Consider a typical web page:

Markup consists of:

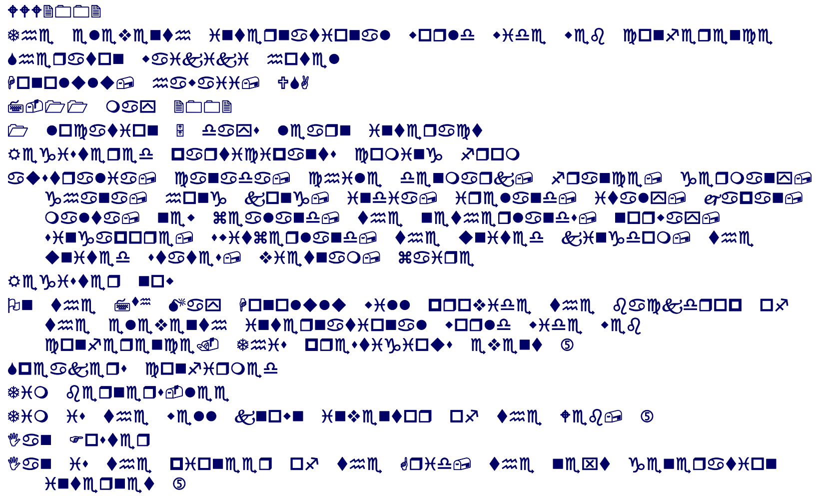
- rendering information (e.g., font size and colour)
- Hyper-links to related content

Semantic content is accessible to humans but not (easily) to computers…

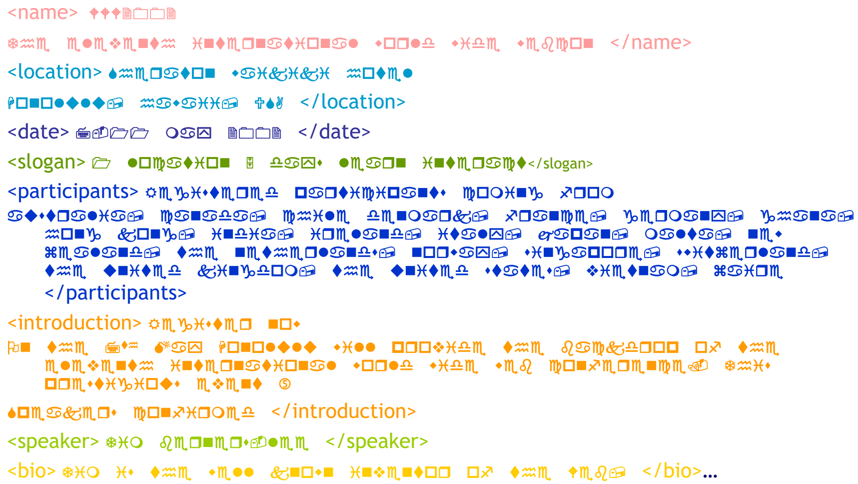
What information can we see…

WWW2002
The eleventh international world wide web conference
Sheraton waikiki hotel
Honolulu, hawaii, USA
7-11 may 2002
1 location 5 days learn interact
Registered participants coming from
australia, canada, chile denmark, france, germany, ghana, hong kong, india, ireland, italy, japan, malta, new zealand, the netherlands, norway, singapore, switzerland, the united kingdom, the united states, vietnam, zaire
Register now
On the 7th May Honolulu will provide the backdrop of the eleventh international world wide web conference. This prestigious event …
Speakers confirmed
Tim berners-lee
Tim is the well known inventor of the Web, …
Ian Foster
Ian is the pioneer of the Grid, the next generation internet …

What information can a machine see…



Solution: XML markup with “meaningful” tags?



But What About…

<conf> [illegible]

[illegible] </conf>

<place> [illegible]

[illegible] </place>

<date> [illegible] </date>

<slogan> [illegible] </slogan>

<participants> [illegible]

</participants>

<introduction> [illegible]

[illegible] </introduction>

<speaker> [illegible] </speaker>

<bio> [illegible]…

Machine sees…

[illegible]

Ontology/KR languages aim to model (part of) world

Terms in language correspond to entities in world

Meaning given by, e.g.:

- Mapping to another formalism, such as FOL, with own well defined semantics
- or a Model Theory (MT)

MT defines relationship between syntax and *interpretations*

- There can be many interpretations (models) of one piece of syntax
- Models supposed to be analogue of (part of) world
  - E.g., elements of model correspond to objects in world
- Formal relationship between syntax and models
  - Structure of models reflect relationships specified in syntax
- Inference (e.g., subsumption) defined in terms of MT
  - E.g., $\mathcal{T} \models A \sqsubseteq B$ iff in every model of $\mathcal{T}$, ext(A) $\subseteq$ ext(B)

Many logics (including standard First Order Logic) use a model theory based on (Zermelo-Frankel) set theory

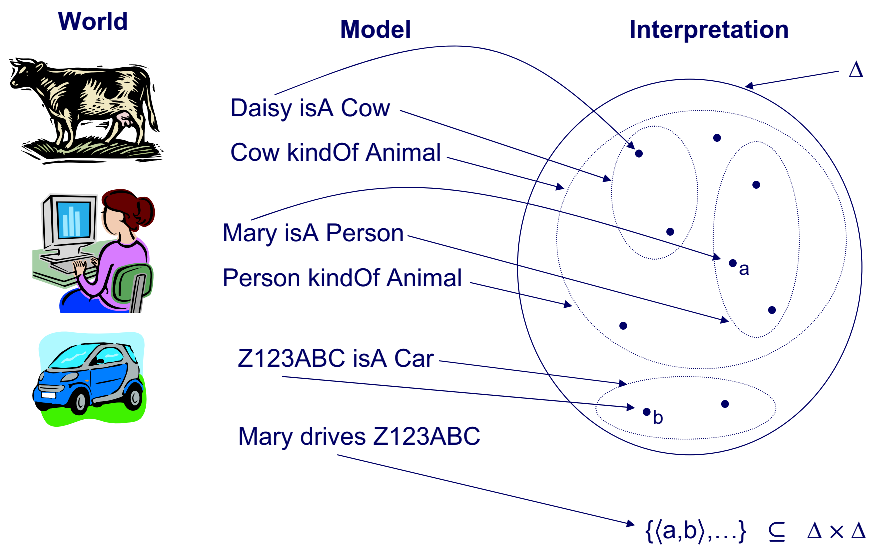
The domain of discourse (i.e., the part of the world being modelled) is represented as a set (often refered as Δ)

Objects in the world are interpreted as elements of Δ

- Classes/concepts (unary predicates) are subsets of Δ
- Properties/roles (binary predicates) are subsets of $\Delta \times \Delta$ (i.e., Δ^2)
- Ternary predicates are subsets of Δ^3 etc.

The sub-class relationship between classes can be interpreted as set inclusion.

Recall: Logics and Model Theory



Recall: Logics and Model Theory

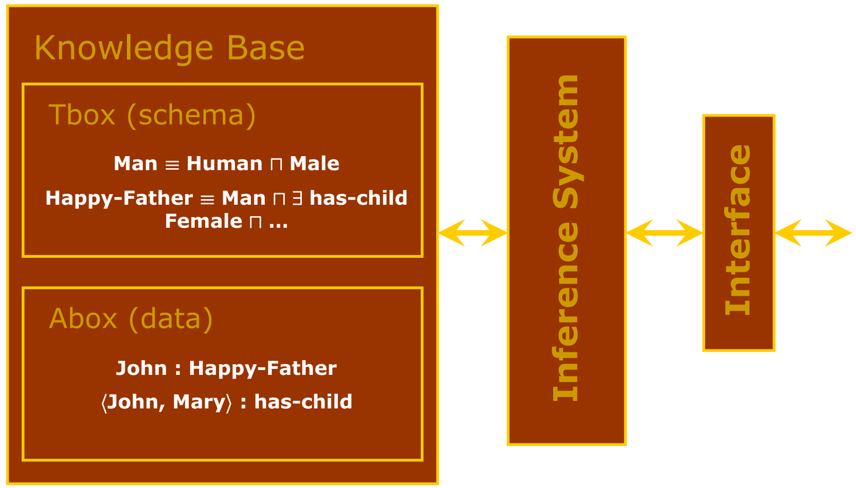
Formally, the vocabulary is the set of names we use in our model of (part of) the world

- {Daisy, Cow, Animal, Mary, Person, Z123ABC, Car, drives, ...}

An interpretation $\mathcal{I}$ is a tuple $\langle \Delta, \cdot^{\mathcal{I}} \rangle$

- Δ is the domain (a set)
- $\cdot^{\mathcal{I}}$ is a mapping that maps
  - **Names of objects to elements of Δ**
  - **Names of unary predicates (classes/concepts) to subsets of Δ**
  - **Names of binary predicates (properties/roles) to subsets of $\Delta \times \Delta$**
  - **And so on for higher arity predicates (if any)**

DL Architecture



DL Knowledge Base

DL Knowledge Base (KB) normally separated into 2 parts:

- TBox is a set of axioms describing structure of domain (i.e., a conceptual schema), e.g.:
  - HappyFather ≡ Man ∧ ∃hasChild.Female ∧ …
  - Elephant ≡ Animal ∧ Large ∧ Grey
  - transitive(ancestor)
- ABox is a set of axioms describing a concrete situation (data), e.g.:
  - John:HappyFather
  - <John,Mary>:hasChild

Separation has no logical significance

- But may be conceptually and implementationally convenient

DL Semantics

Interpretation function $\cdot^{\mathcal{I}}$ extends to concept expressions in the obvious way, i.e.:

$$
\begin{aligned}
(C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} &= C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
(\neg C)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \\
\{x\}^{\mathcal{I}} &= \{x^{\mathcal{I}}\} \\
(\exists R.C)^{\mathcal{I}} &= \{x \mid \exists y.\langle x, y\rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\} \\
(\forall R.C)^{\mathcal{I}} &= \{x \mid \forall y.(x, y) \in R^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\} \\
(\leqslant nR)^{\mathcal{I}} &= \{x \mid \#\{y \mid \langle x, y\rangle \in R^{\mathcal{I}}\} \leqslant n\} \\
(\geqslant nR)^{\mathcal{I}} &= \{x \mid \#\{y \mid \langle x, y\rangle \in R^{\mathcal{I}}\} \geqslant n\}
\end{aligned}
$$

DL Knowledge Bases (Ontologies)

A DL Knowledge Base is of the form $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$

- $\mathcal{T}$ (Tbox) is a set of axioms of the form:
  - $C \sqsubseteq D$ (concept inclusion)
  - $C \equiv D$ (concept equivalence)
  - $R \sqsubseteq S$ (role inclusion)
  - $R \equiv S$ (role equivalence)
  - $R^+ \sqsubseteq R$ (role transitivity)
- $\mathcal{A}$ (Abox) is a set of axioms of the form
  - $x \in D$ (concept instantiation)
  - $\langle x,y \rangle \in R$ (role instantiation)

Two sorts of Tbox axioms often distinguished

- "Definitions"
  - $C \sqsubseteq D$ or $C \equiv D$ where C is a concept name
- General Concept Inclusion axioms (GCIs)
  - $C \sqsubseteq D$ where C is an arbitrary concept

Knowledge Base Semantics

An interpretation $\mathcal{I}$ satisfies (models) an axiom A ($\mathcal{I} \vDash A$):

- $\mathcal{I} \vDash C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
- $\mathcal{I} \vDash C \equiv D$ iff $C^{\mathcal{I}} = D^{\mathcal{I}}$
- $\mathcal{I} \vDash R \sqsubseteq S$ iff $R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$
- $\mathcal{I} \vDash R \equiv S$ iff $R^{\mathcal{I}} = S^{\mathcal{I}}$
- $\mathcal{I} \vDash R^+ \sqsubseteq R$ iff $(R^{\mathcal{I}})^+ \subseteq R^{\mathcal{I}}$
- $\mathcal{I} \vDash x \in D$ iff $x^{\mathcal{I}} \in D^{\mathcal{I}}$
- $\mathcal{I} \vDash \langle x,y \rangle \in R$ iff $(x^{\mathcal{I}},y^{\mathcal{I}}) \in R^{\mathcal{I}}$

$\mathcal{I}$ satisfies a Tbox $\mathcal{T}$ ($\mathcal{I} \vDash \mathcal{T}$) iff $\mathcal{I}$ satisfies every axiom A in $\mathcal{T}$

$\mathcal{I}$ satisfies an Abox $\mathcal{A}$ ($\mathcal{I} \vDash \mathcal{A}$) iff $\mathcal{I}$ satisfies every axiom A in $\mathcal{A}$

$\mathcal{I}$ satisfies an KB $\mathcal{K}$ ($\mathcal{I} \vDash \mathcal{K}$) iff $\mathcal{I}$ satisfies both $\mathcal{T}$ and $\mathcal{A}$

Inference Tasks

Knowledge is correct (captures intuitions)

- C subsumes D w.r.t. $\mathcal{K}$ iff for *every* model $\mathcal{I}$ of $\mathcal{K}$, $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$

Knowledge is minimally redundant (no unintended synonyms)

- C is equivalent to D w.r.t. $\mathcal{K}$ iff for *every* model $\mathcal{I}$ of $\mathcal{K}$, $C^{\mathcal{I}} = D^{\mathcal{I}}$

Knowledge is meaningful (classes can have instances)

- C is satisfiable w.r.t. $\mathcal{K}$ iff there exists *some* model $\mathcal{I}$ of $\mathcal{K}$ s.t. $C^{\mathcal{I}} \neq \emptyset$

Querying knowledge

- x is an instance of C w.r.t. $\mathcal{K}$ iff for *every* model $\mathcal{I}$ of $\mathcal{K}$, $x^{\mathcal{I}} \in C^{\mathcal{I}}$
- $\langle x,y \rangle$ is an instance of R w.r.t. $\mathcal{K}$ iff for, *every* model $\mathcal{I}$ of $\mathcal{K}$, $(x^{\mathcal{I}},y^{\mathcal{I}}) \in R^{\mathcal{I}}$

Knowledge base consistency

- A KB $\mathcal{K}$ is consistent iff there exists *some* model $\mathcal{I}$ of $\mathcal{K}$

Syntax für DLs (ohne concrete domains)

AIFB

	Concepts	
ALC	Atomic	A, B
ALC	Not	$\neg$C
ALC	And	C $\sqcap$ D
ALC	Or	C $\sqcup$ D
ALC	Exists	$\exists$R.C
ALC	For all	$\forall$R.C
Q (N)	At least	$\geq$n R.C ($\geq$n R)
Q (N)	At most	$\leq$n R.C ($\leq$n R)
O	Nominal	$\{i_1,\ldots,i_n\}$

	Roles	
I	Atomic	R
I	Inverse	R^-

Ontology (=Knowledge Base)

	Concept Axioms (TBox)	
	Subclass	C $\sqsubseteq$ D
	Equivalent	C $\equiv$ D

	Role Axioms (RBox)	
H	Subrole	R $\sqsubseteq$ S
S	Transitivity	Trans(S)

Assertional Axioms (ABox)	
Instance	C(a)
Role	R(a,b)
Same	a = b
Different	a $\neq$ b

S = ALC + Transitivity **OWL DL = SHOIN(D)** (D: concrete domain)

Examples

- Person $\sqcap$ Female
- Person $\sqcap$ $\exists$attends.Course
- Person $\sqcap$ $\forall$attends.(Course $\to$ $\neg$Easy)
- Person $\sqcap$ $\exists$teaches.(Course $\sqcap$ $\forall$attended-by.(Bored $\sqcup$ Sleeping))

The Description Logic $\mathcal{ALC}$: Syntax

Atomic types: concept names $A, B, \ldots$ (unary predicates)

role names $R, S, \ldots$ (binary predicates)

Constructors:

- $\neg C$ (negation)
- $C \sqcap D$ (conjunction)
- $C \sqcup D$ (disjunction)
- $\exists R.C$ (existential restriction)
- $\forall R.C$ (value restriction)

Abbreviations:

- $C \to D = \neg C \sqcup D$ (implication)
- $C \leftrightarrow D = C \to D \sqcap D \to C$ (bi-implication)
- $\top = (A \sqcup \neg A)$ (top concept)
- $\bot = A \sqcap \neg A$ (bottom concept)

Interpretations

Semantics based on interpretations $(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where

- $\Delta^{\mathcal{I}}$ is a non-empty set (the domain)
- $\cdot^{\mathcal{I}}$ is the interpretation function mapping

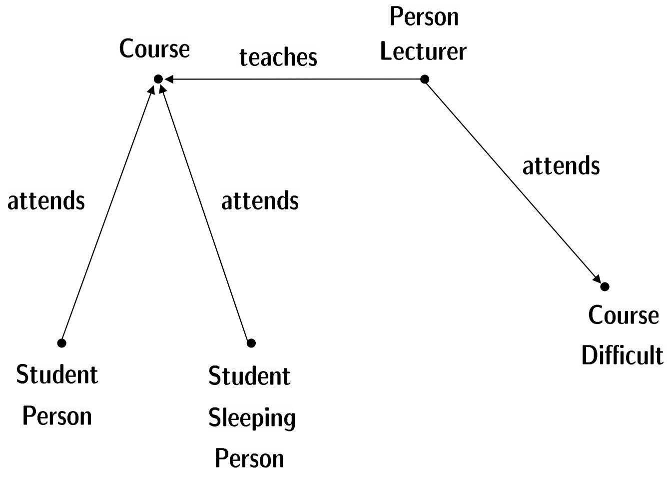
  each concept name A to a subset $A^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$ and

  each role name R to a binary relation $R^{\mathcal{I}}$ over $\Delta^{\mathcal{I}}$.

Intuition: interpretation is complete description of the world

Technically: interpretation is first-order structure
with only unary and binary predicates

Example

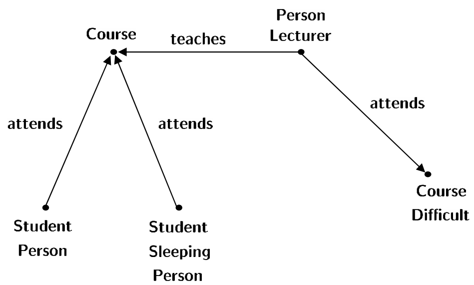


Semantics of Complex Concepts

$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$ $\quad (C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$ $\quad (C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$

$(\exists R.C)^{\mathcal{I}} = \{d \mid \text{there is an } e \in \Delta^{\mathcal{I}} \text{ with } (d,e) \in R^{\mathcal{I}} \text{ and } e \in C^{\mathcal{I}}\}$

$(\forall R.C)^{\mathcal{I}} = \{d \mid \text{for all } e \in \Delta^{\mathcal{I}}, (d,e) \in R^{\mathcal{I}} \text{ implies } e \in C^{\mathcal{I}}\}$



Person ⊓ ∃attends.Course

Person ⊓ ∀attends.(¬Course ⊔ Difficult)

TBoxes

Capture an application's terminology means defining concepts

TBoxes are used to store concept definitions:

Syntax:

finite set of concept equations $A \doteq C$

with A concept name and C concept

left-hand sides must be unique!

Semantics:

interpretation $\mathcal{I}$ satisfies $A \doteq C$ iff $A^{\mathcal{I}} = C^{\mathcal{I}}$

$\mathcal{I}$ is model of $\mathcal{T}$ if it satisfies all definitions in $\mathcal{T}$

E.g.: Lecturer ≐ Person ⊓ ∃teaches.Course

Yields two kinds of concept names: defined and primitive

TBox: Example

TBoxes are used as ontologies:

Woman ≐ Person ⊓ Female

Man ≐ Person ⊓ ¬Woman

Lecturer ≐ Person ⊓ ∃teaches.Course

Student ≐ Person ⊓ ∃attends.Course

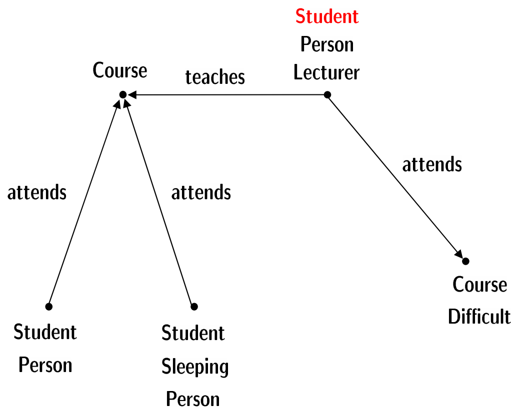
BadLecturer ≐ Person ⊓ ∀teaches.(Course → Boring)

TBox: Example II

A TBox restricts the set of admissible interpretations.

$$\text{Lecturer} \doteq \text{Person} \sqcap \exists \text{teaches.Course}$$

$$\text{Student} \doteq \text{Person} \sqcap \exists \text{attends.Course}$$



Reasoning Tasks — Subsumption

C subsumed by D w.r.t. $\mathcal{T}$ (written $C \sqsubseteq_{\mathcal{T}} D$)

iff

$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ holds for all models $\mathcal{I}$ of $\mathcal{T}$

Intuition: If $C \sqsubseteq_{\mathcal{T}} D$, then D is more general than C

Example:

$$\text{Lecturer} \doteq \text{Person} \sqcap \exists \text{teaches.Course}$$

$$\text{Student} \doteq \text{Person} \sqcap \exists \text{attends.Course}$$

Then

$$\text{Lecturer} \sqcap \exists \text{attends.Course} \sqsubseteq_{\mathcal{T}} \text{Student}$$

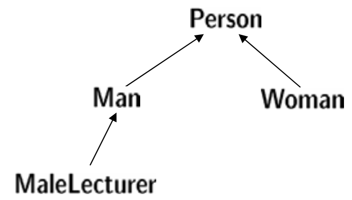
Reasoning Tasks — Classification

Classification: arrange all defined concepts from a TBox in a hierarchy w.r.t. generality

$$\text{Woman} \doteq \text{Person} \sqcap \text{Female}$$

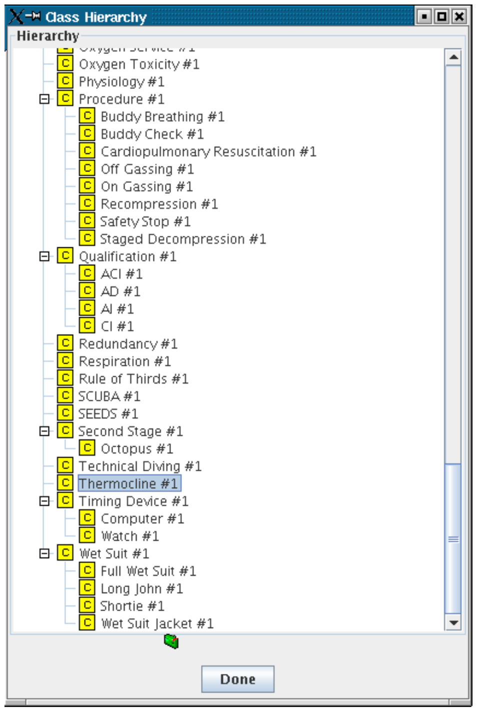
$$\text{Man} \doteq \text{Person} \sqcap \neg\text{Woman}$$

$$\text{MaleLecturer} \doteq \text{Man} \sqcap \exists \text{teaches.Course}$$



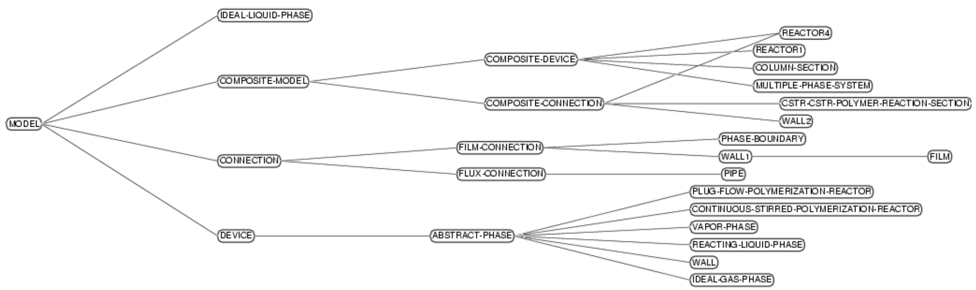
Can be computed using multiple subsumption tests

Provides a principled view on ontology for browsing, maintaining, etc.



A Concept Hierarchy

Excerpt from a process engineering ontology



Reasoning Tasks — Satisfiability

C is satisfiable w.r.t. $\mathcal{T}$ iff $\mathcal{T}$ has a model with $C^{\mathcal{I}} \neq \emptyset$

Intuition: If unsatisfiable, the concept contains a contradiction.

Example: Woman $\doteq$ Person $\sqcap$ Female

Man $\doteq$ Person $\sqcap$ $\neg$Woman

Then $\exists$sibling.Man $\sqcap$ $\forall$sibling.Woman is unsatisfiable w.r.t. $\mathcal{T}$

Subsumption can be reduced to (un)satisfiability and vice versa:

- $C \sqsubseteq_{\mathcal{T}} D$ iff $C \sqcap \neg D$ is not satisfiable w.r.t. $\mathcal{T}$
- C is satisfiable w.r.t. $\mathcal{T}$ if not $C \sqsubseteq_{\mathcal{T}} \bot$.

Many reasoners decide satisfiability rather than subsumption.

Definitorial TBoxes

A primitive interpretation for TBox $\mathcal{T}$ interpretes

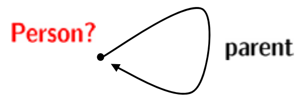
- the primitive concept names in $\mathcal{T}$
- all role names

A TBox is called definitorial if every primitive interpretation for $\mathcal{T}$ can be uniquely extended to a model of $\mathcal{T}$.

i.e.: primitive concepts (and roles) uniquely determine defined concepts

Not all TBoxes are definitorial:

Person $\doteq$ $\exists$parent.Person



Non-definitorial TBoxes describe constraints, e.g. from background knowledge

Acyclic TBoxes

TBox $\mathcal{T}$ is acyclic if there are no definitorial cycles:

~~Lecturer $\doteq$ Person $\sqcap$ $\exists$teaches.Course~~

~~Course $\doteq$ $\exists$has-title.Title $\sqcap$ $\exists$tought-by.Lecturer~~

Expansion of acyclic TBox $\mathcal{T}$:

exhaustively replace defined concept names with their definition

(terminates due to acyclicity)

Acyclic TBoxes are always definitorial:

first expand, then set $A^{\mathcal{I}} := C^{\mathcal{I}}$ for all $A \doteq C \in \mathcal{T}$

Acyclic TBoxes II

For reasoning, acyclic TBox can be eliminated:

- to decide $C \sqsubseteq_{\mathcal{T}} D$ with $\mathcal{T}$ acyclic,
  - expand $\mathcal{T}$
  - replace defined concept names in C, D with their definition
  - decide $C \sqsubseteq D$
- analogously for satisfiability

May yield an exponential blow-up:

$$A_0 \doteq \forall r.A_1 \sqcap \forall s.A_1$$
$$A_1 \doteq \forall r.A_2 \sqcap \forall s.A_2$$
$$\cdots$$
$$A_{n-1} \doteq \forall r.A_n \sqcap \forall s.A_n$$

General Concept Inclusions

View of TBox as set of constraints

General TBox: finite set of general concept implications (GCIs)

$$C \sqsubseteq D$$

with both C and D allowed to be complex

e.g. Course $\sqcap$ $\forall$attended-by.Sleeping $\sqsubseteq$ Boring

Interpretation $\mathcal{I}$ is model of general TBox $\mathcal{T}$ if

$$C^{\mathcal{I}} \subseteq D^{\mathcal{I}} \text{ for all } C \sqsubseteq D \in \mathcal{T}.$$

$C \doteq D$ is abbreviation for $C \sqsubseteq D$, $D \sqsubseteq C$

e.g. Student $\sqcap$ $\exists$has-favourite.SoccerTeam $\doteq$ Student $\sqcap$ $\exists$has-favourite.Beer

Note: $C \sqsubseteq D$ equivalent to $\top \doteq C \rightarrow D$

ABoxes

ABoxes describe a snapshot of the world

An ABox is a finite set of assertions

$a : C$	(a individual name, C concept)
$(a, b) : R$	(a, b individual names, R role name)

E.g. {peter : Student, (dl-course, uli) : tought-by}

Interpretations $\mathcal{I}$ map each individual name a to an element of $\Delta^{\mathcal{I}}$.

$\mathcal{I}$ satisfies an assertion

$a : C$	iff	$a^{\mathcal{I}} \in C^{\mathcal{I}}$
$(a, b) : R$	iff	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$

$\mathcal{I}$ is a model for an ABox $\mathcal{A}$ if $\mathcal{I}$ satisfies all assertions in $\mathcal{A}$.

ABoxes II

Note:

- interpretations describe the state if the world in a complete way
- ABoxes describe the state if the world in an incomplete way

(uli, dl-course) : tought-by uli : Female

does not imply

dl-course : $\forall$tought-by.Female

An ABox has many models!

An ABox constraints the set of admissibile models similar to a TBox

Reasoning with ABoxes

ABox consistency

Given an ABox $\mathcal{A}$ and a TBox $\mathcal{T}$, do they have a common model?

Instance checking

Given an ABox $\mathcal{A}$, a TBox $\mathcal{T}$, an individual name a, and a concept C does $a^{\mathcal{I}} \in C^{\mathcal{I}}$ hold in all models of $\mathcal{A}$ and $\mathcal{T}$?

(written $\mathcal{A}, \mathcal{T} \models a : C$)

The two tasks are interreducible:

- $\mathcal{A}$ consistent w.r.t. $\mathcal{T}$ iff $\mathcal{A}, \mathcal{T} \not\models a : \bot$
- $\mathcal{A}, \mathcal{T} \models a : C$ iff $\mathcal{A} \cup \{a : \neg C\}$ is not consistent

Example for ABox Reasoning

ABox

dumbo : Mammal

~~g23 : Darkgrey~~

(dumbo, g23) : color

dumbo : ∀color.Lightgrey

t14 : Trunk

(dumbo, t14) : bodypart

TBox

Elephant $\doteq$ Mammal ⊓ ∃bodypart.Trunk ⊓ ∀color.Grey

Grey $\doteq$ Lightgrey ⊔ Darkgrey

$\bot \doteq$ Lightgrey ⊓ Darkgrey

1. ABox is inconsistent w.r.t. TBox.
2. dumbo is an instance of Elephant

2. Tableau algorithms for $\mathcal{ALC}$ and extensions

We see a tableau algorithm for $\mathcal{ALC}$ and extend it with

① general TBoxes and

② inverse roles

Goal: Design sound and complete desicion procedures for satisfiability (and subsumption) of DLs which are well-suited for implementation purposes

A tableau algorithm for the satisfiability of $\mathcal{ALC}$ concepts

Goal: design an algorithm which takes an $\mathcal{ALC}$ concept C_0 and

1. returns *"satisfiable"* iff C_0 is satisfiable and
2. terminates, on every input,

i.e., which **decides** satisfiability of $\mathcal{ALC}$ concepts.

Recall: such an algorithm **cannot** exist for FOL since satisfiability of FOL is undecidable.

Idea: our algorithm

- is tableau-based and
- tries to construct a **model** of C_0
- by breaking C_0 down syntactically, thus
- inferring new constraints on such a model.

Preliminaries: Negation Normal Form

To make our life easier, we transform each concept C_0 into an **equivalent** C_1 in **NNF**

Equivalent: $C_0 \sqsubseteq C_1$ and $C_1 \sqsubseteq C_0$

NNF: negation occurs only in front of concept names

How? By pushing negation inwards (de Morgan et. al):

$$\begin{aligned}\neg(C \sqcap D) &\rightsquigarrow \neg C \sqcup \neg D\\ \neg(C \sqcup D) &\rightsquigarrow \neg C \sqcap \neg D\\ \neg\neg C &\rightsquigarrow C\\ \neg\forall R.C &\rightsquigarrow \exists R.\neg C\\ \neg\exists R.C &\rightsquigarrow \forall R.\neg C\end{aligned}$$

From now on: concepts are in NNF and
sub(C) denotes the set of all sub-concepts of C

More intuition

Find out whether $A \sqcap \exists R.B \sqcap \forall R.\neg B$ is satisfiable...
$A \sqcap \exists R.B \sqcap \forall R.(\neg B \sqcup \exists S.E)$

Our tableau algorithm works on a **completion tree** which

- represents a model $\mathcal{I}$: **nodes** represent elements of $\Delta^{\mathcal{I}}$
  $\rightsquigarrow$ each node x is labelled with concepts $\mathcal{L}(x) \subseteq$ **sub**(C_0)
  $C \in \mathcal{L}(x)$ is read as "x should be an instance of C"
  edges represent role successorship
  $\rightsquigarrow$ each edge $\langle x, y\rangle$ is labelled with a role-name from C_0
  $R \in \mathcal{L}(\langle x, y\rangle)$ is read as "(x, y) should be in $R^{\mathcal{I}}$"
- is initialised with a single root node x_0 with $\mathcal{L}(x_0) = \{C_0\}$
- is expanded using **completion rules**

Completion rules for $\mathcal{ALC}$

$\sqcap$-rule: if $C_1 \sqcap C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \not\subseteq \mathcal{L}(x)$
then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C_1, C_2\}$

$\sqcup$-rule: if $C_1 \sqcup C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \cap \mathcal{L}(x) = \emptyset$
then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C\}$ for some $C \in \{C_1, C_2\}$

$\exists$-rule: if $\exists S.C \in \mathcal{L}(x)$ and x has no S-successor y with $C \in \mathcal{L}(y)$,
then create a new node y with $\mathcal{L}(\langle x, y\rangle) = \{S\}$ and $\mathcal{L}(y) = \{C\}$

$\forall$-rule: if $\forall S.C \in \mathcal{L}(x)$ and there is an S-successor y of x with $C \notin \mathcal{L}(y)$
then set $\mathcal{L}(y) = \mathcal{L}(y) \cup \{C\}$

Properties of the completion rules for $\mathcal{ALC}$

We only apply rules if their application does **"something new"**

$\sqcap$-rule: if $C_1 \sqcap C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \not\subseteq \mathcal{L}(x)$
then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C_1, C_2\}$

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Properties of the completion rules for $\mathcal{ALC}$

The $\sqcup$-rule is **non-deterministic:**

$\sqcap$-rule: if $C_1 \sqcap C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \not\subseteq \mathcal{L}(x)$
then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C_1, C_2\}$

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Last details on tableau algorithm for $\mathcal{ALC}$

Clash: a c-tree contains a **clash** if it has a node x with $\bot \in \mathcal{L}(x)$ or $\{A, \neg A\} \subseteq \mathcal{L}(x)$ — otherwise, it is **clash-free**

Complete: a c-tree is **complete** if none of the completion rules can be applied to it

Answer behaviour: when started for C_0 (in NNF!), the tableau algorithm

- is **initialised** with a single root node x_0 with $\mathcal{L}(x_0) = \{C_0\}$
- repeatedly applies the **completion rules** (in whatever order it likes)
- **answer** "*C_0 is satisfiable*" iff the completion rules **can** be applied **in such a way that** it results in a complete and clash-free c-tree (careful: this is non-deterministic)

...go back to examples

Properties of our tableau algorithm

Lemma: Let C_0 an $\mathcal{ALC}$-concept in NNF. Then

1. the algorithm terminates when applied to C_0 and
2. the rules can be applied such that they generate a clash-free and complete completion tree iff C_0 is satisfiable.

Corollary:

1. Our tableau algorithm **decides satisfiability** and **subsumption** of $\mathcal{ALC}$.
2. Satisfiability (and subsumption) in $\mathcal{ALC}$ is decidable in **PSpace**.
3. $\mathcal{ALC}$ has the **finite model property** i.e., every satisfiable concept has a **finite** model.
4. $\mathcal{ALC}$ has the **tree model property** i.e., every satisfiable concept has a **tree** model.
5. $\mathcal{ALC}$ has the **finite tree model property** i.e., every satisfiable concept has a **finite tree** model.

Extend tableau algorithm to $\mathcal{ALC}$ with general TBoxes

Recall:

- **Concept inclusion:** of the form $C \dot{\sqsubseteq} D$ for C, D (complex) concepts
- **(General) TBox:** a finite set of concept inclusions
- $\mathcal{I}$ **satisfies** $C \dot{\sqsubseteq} D$ iff $C^\mathcal{I} \subseteq D^\mathcal{I}$
- $\mathcal{I}$ is **a model of TBox** $\mathcal{T}$ iff $\mathcal{I}$ satisfies each concept equation in $\mathcal{T}$
- C_0 is **satisfiable w.r.t.** $\mathcal{T}$ iff there is a model $\mathcal{I}$ of $\mathcal{T}$ with $C_0^\mathcal{I} \neq \emptyset$

Goal – Lemma: Let C_0 an $\mathcal{ALC}$-concept and $\mathcal{T}$ be a an $\mathcal{ALC}$-TBox. Then

1. the algorithm terminates when applied to $\mathcal{T}$ and C_0 and
2. the rules can be applied such that they generate a clash-free and complete completion tree iff C_0 is satisfiable w.r.t. $\mathcal{T}$.

Extend tableau algorithm to $\mathcal{ALC}$ with general TBoxes: Preliminaries

We extend our tableau algorithm by adding a **new completion rule**:

- remember that nodes represent elements of $\Delta^{\mathcal{I}}$ and
- if $C \dot\sqsubseteq D \in \mathcal{T}$, then for each element x in a model $\mathcal{I}$ of $\mathcal{T}$
  - if $x \in C^{\mathcal{I}}$, then $x \in D^{\mathcal{I}}$
  - hence $x \in (\neg C)^{\mathcal{I}}$ or $x \in D^{\mathcal{I}}$
  - $x \in (\neg C \sqcup D)^{\mathcal{I}}$
  - $x \in (\mathsf{NNF}(\neg C \sqcup D))^{\mathcal{I}}$

  for $\mathsf{NNF}(E)$ the negation normal form of E

Completion rules for $\mathcal{ALC}$ with TBoxes

$\sqcap$-rule: if $C_1 \sqcap C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \not\subseteq \mathcal{L}(x)$
then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C_1, C_2\}$

$\sqcup$-rule: if $C_1 \sqcup C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \cap \mathcal{L}(x) = \emptyset$
then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C\}$ for some $C \in \{C_1, C_2\}$

$\exists$-rule: if $\exists S.C \in \mathcal{L}(x)$ and x has no S-successor y with $C \in \mathcal{L}(y)$,
then create a new node y with $\mathcal{L}(\langle x, y\rangle) = \{S\}$ and $\mathcal{L}(y) = \{C\}$

$\forall$-rule: if $\forall S.C \in \mathcal{L}(x)$ and there is an S-successor y of x with $C \notin \mathcal{L}(y)$
then set $\mathcal{L}(y) = \mathcal{L}(y) \cup \{C\}$

$\mathcal{T}$-rule: if $C_1 \dot\sqsubseteq C_2 \in \mathcal{T}$ and $\mathsf{NNF}(\neg C_1 \sqcup C_2) \notin \mathcal{L}(x)$
then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{\mathsf{NNF}(\neg C_1 \sqcup C_2)\}$

A tableau algorithm for $\mathcal{ALC}$ with general TBoxes

Example: Consider satisfiability of C w.r.t. $\{C \dot\sqsubseteq \exists R.C\}$

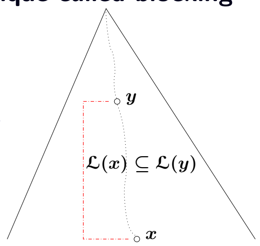
Tableau algorithm no longer terminates!

Reason: size of concepts no longer decreases along paths in a completion tree

Observation: most nodes on this path look the same and we keep repeating ourselves

Regain termination with a "cycle-detection" technique called blocking

Intuitively, whenever we find a situation where y has to satisfy *stronger* constraints than x, we *freeze* x, i.e., block rules from being applied to x



A tableau algorithm for $\mathcal{ALC}$ with general TBoxes: Blocking

- x is **directly blocked** if it has an ancestor y with $\mathcal{L}(x) \subseteq \mathcal{L}(y)$
- in this case and if y is the "closest" such node to x, we say that x **is blocked by** y
- a node is **blocked** if it is directly blocked or one of its ancestors is blocked

$\oplus$ restrict the application of all rules to nodes which are not blocked

$\rightsquigarrow$ **completion rules for $\mathcal{ALC}$ w.r.t. TBoxes**

A tableau algorithm for $\mathcal{ALC}$ with general TBoxes

$\sqcap$-rule: if $C_1 \sqcap C_2 \in \mathcal{L}(x)$, $\{C_1, C_2\} \not\subseteq \mathcal{L}(x)$, and x is not blocked
then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C_1, C_2\}$

$\sqcup$-rule: if $C_1 \sqcup C_2 \in \mathcal{L}(x)$, $\{C_1, C_2\} \cap \mathcal{L}(x) = \emptyset$, and x is not blocked
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$\forall$-rule: if $\forall S.C \in \mathcal{L}(x)$, there is an S-successor y of x with $C \notin \mathcal{L}(y)$
and x is not blocked
then set $\mathcal{L}(y) = \mathcal{L}(y) \cup \{C\}$

$\mathcal{T}$-rule: if $C_1 \dot\sqsubseteq C_2 \in \mathcal{T}$, NNF$(\neg C_1 \sqcup C_2) \notin \mathcal{L}(x)$
and x is not blocked
then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{NNF(\neg C_1 \sqcup C_2)\}$

Tableaux Rules for $\mathcal{ALC}$

$x \bullet \{C_1 \sqcap C_2, \ldots\}$	$\rightarrow_\sqcap$	$x \bullet \{C_1 \sqcap C_2, C_1, C_2, \ldots\}$
$x \bullet \{C_1 \sqcup C_2, \ldots\}$	$\rightarrow_\sqcup$	$x \bullet \{C_1 \sqcup C_2, C, \ldots\}$ for $C \in \{C_1, C_2\}$
$x \bullet \{\exists R.C, \ldots\}$	$\rightarrow_\exists$	$x \bullet \{\exists R.C, \ldots\}$ $\xrightarrow{R}$ $y \bullet \{C\}$
$x \bullet \{\forall R.C, \ldots\}$ $\xrightarrow{R}$ $y \bullet \{\ldots\}$	$\rightarrow_\forall$	$x \bullet \{\forall R.C, \ldots\}$ $\xrightarrow{R}$ $y \bullet \{C, \ldots\}$

Tableaux Rule for Transitive Roles

$x \bullet \{\forall R.C, \ldots\}$ $\xrightarrow{R}$ $y \bullet \{\ldots\}$	$\rightarrow_{\forall_+}$	$x \bullet \{\forall R.C, \ldots\}$ $\xrightarrow{R}$ $y \bullet \{\forall R.C, \ldots\}$

Where R is a transitive role (i.e., $(R^\mathcal{I})^+ = R^\mathcal{I}$)

- No longer naturally terminating (e.g., if $C = \exists R.\top$)
- Need blocking
  - Simple blocking suffices for $\mathcal{ALC}$ plus transitive roles
  - I.e., do not expand node label if ancestor has superset label
  - More expressive logics (e.g., with inverse roles) need more sophisticated blocking strategies

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role

$$\mathcal{L}(w) = \{\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$$

w

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role

$$\mathcal{L}(w) = \{\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$$

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Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role

$$\mathcal{L}(w) = \{\exists S.C, \forall S.(\neg C \sqcup \neg D), \exists R.C, \forall R.(\exists R.C)\}$$

w

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Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role

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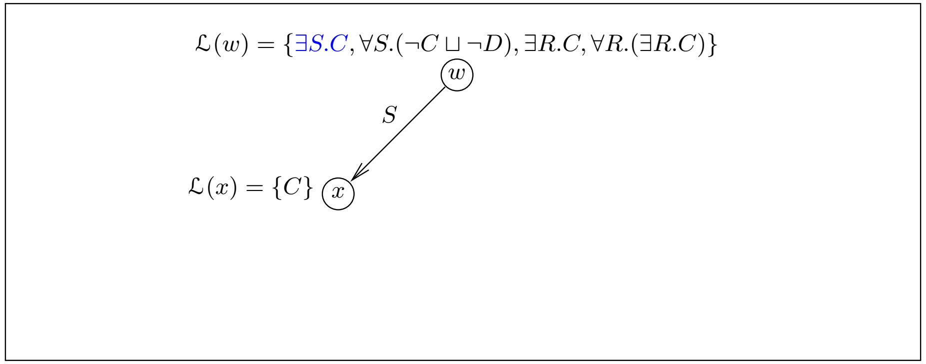

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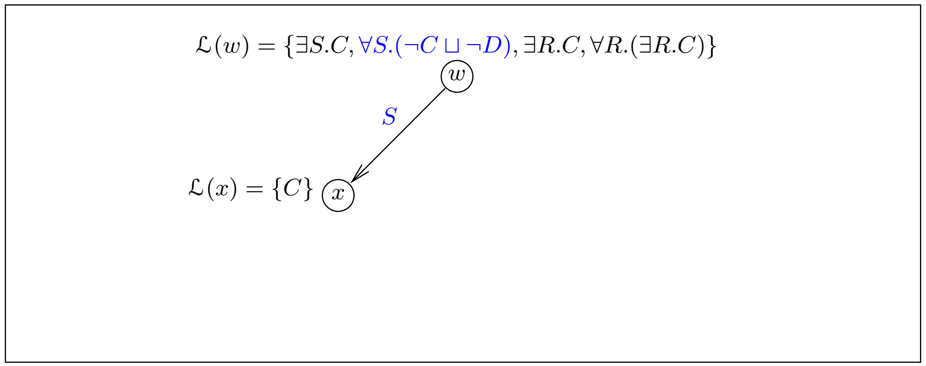

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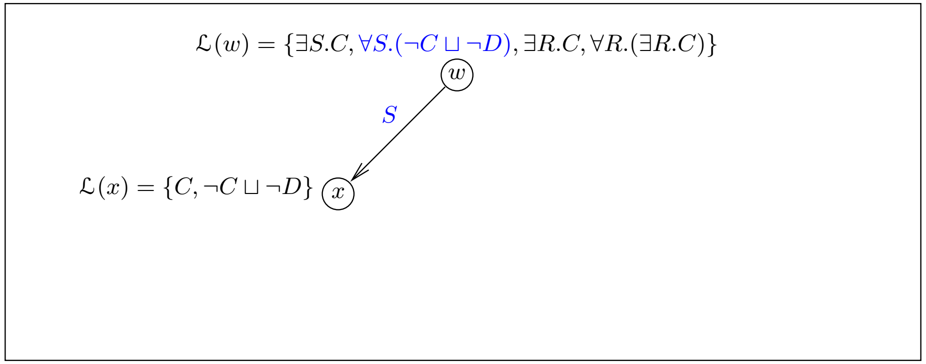

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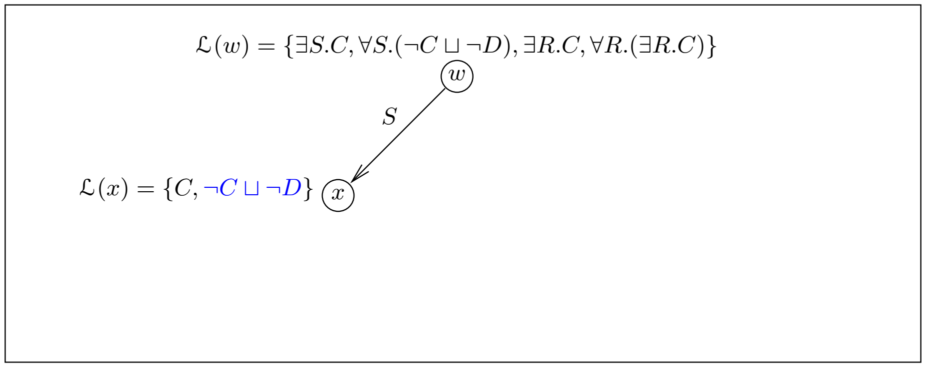

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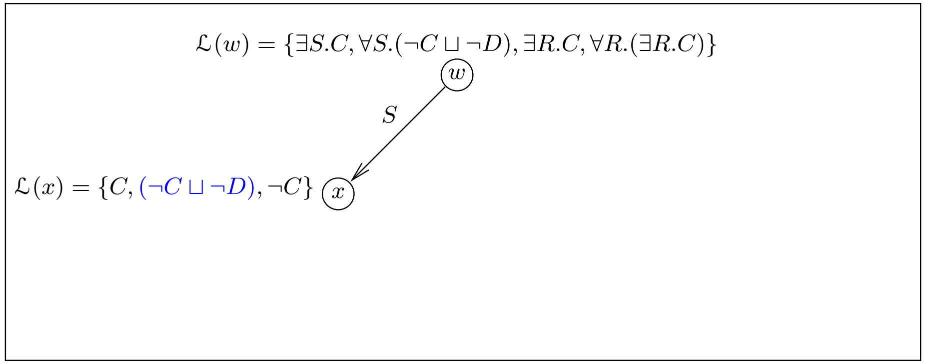

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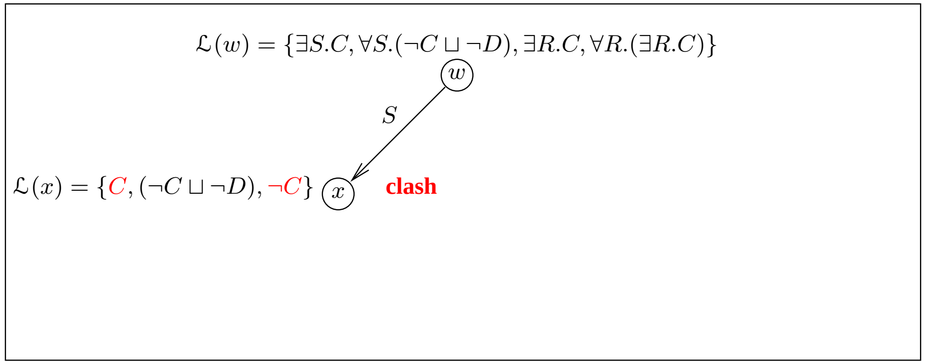

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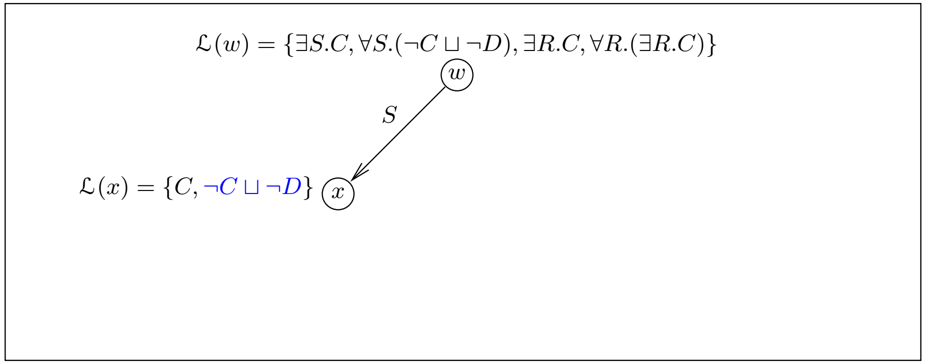

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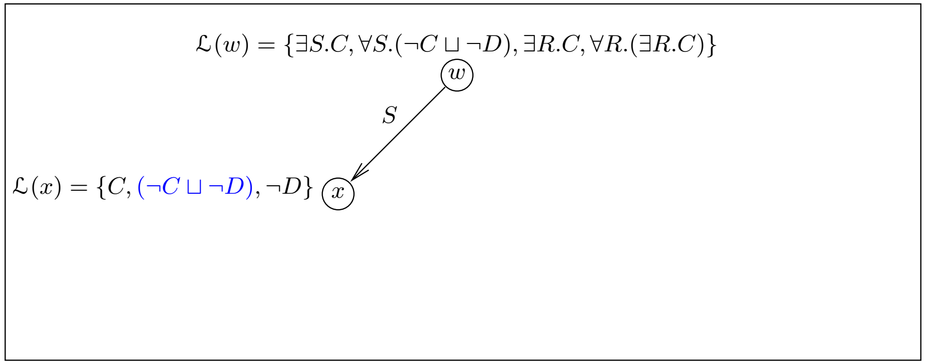

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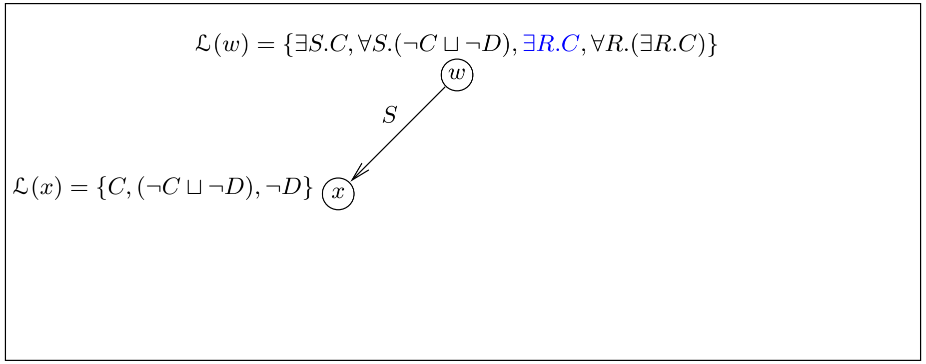

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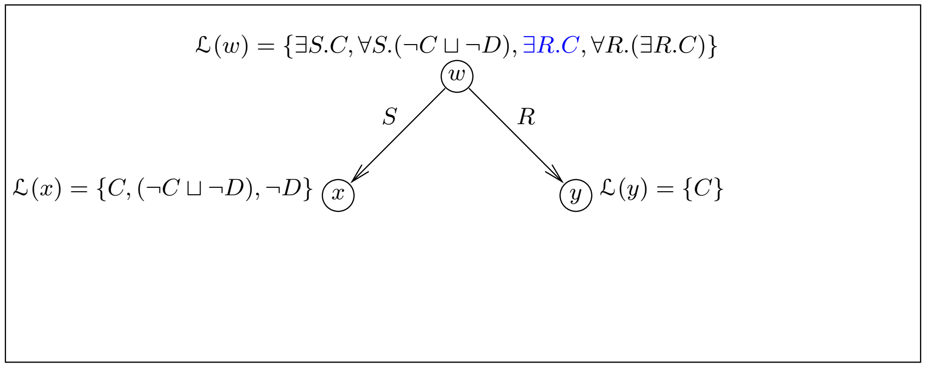

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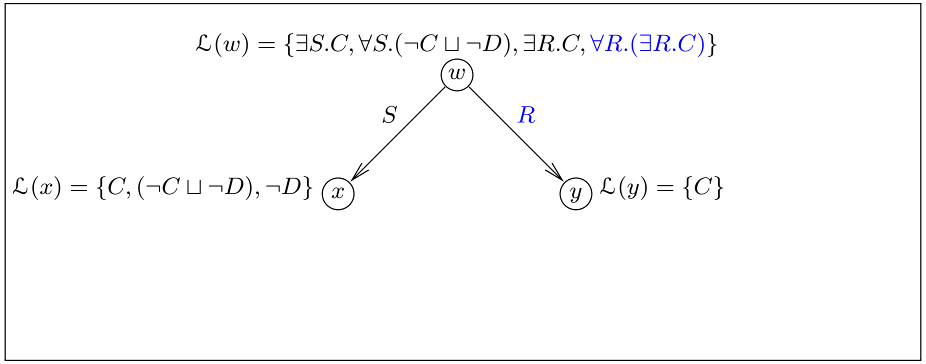

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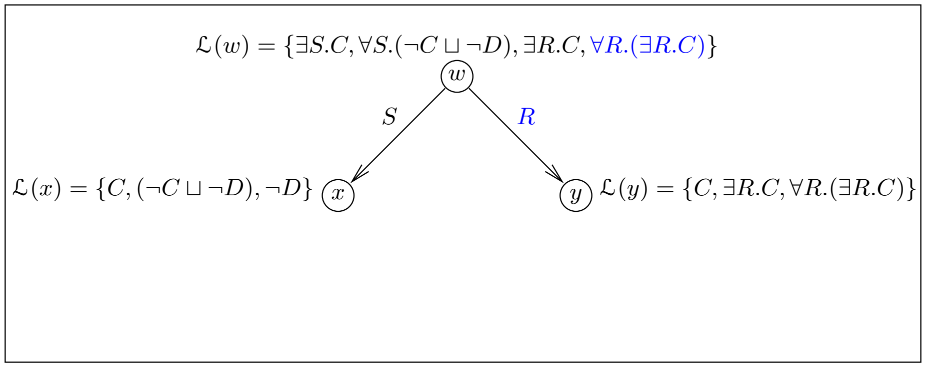

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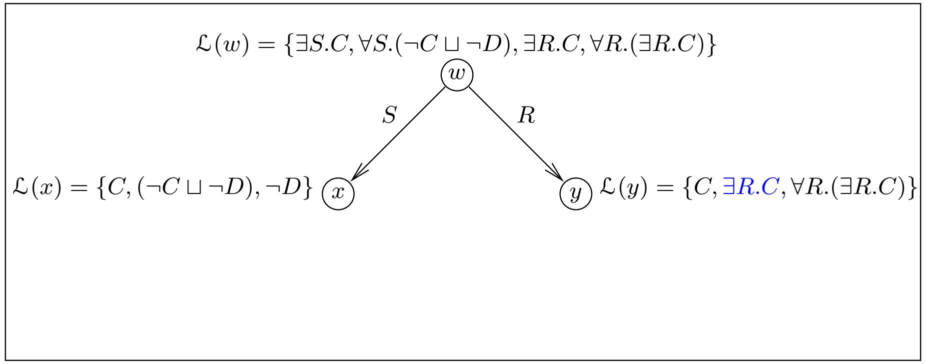

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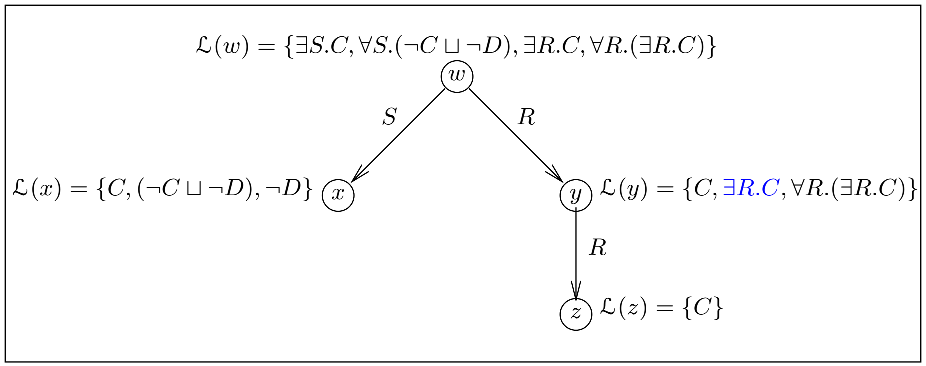

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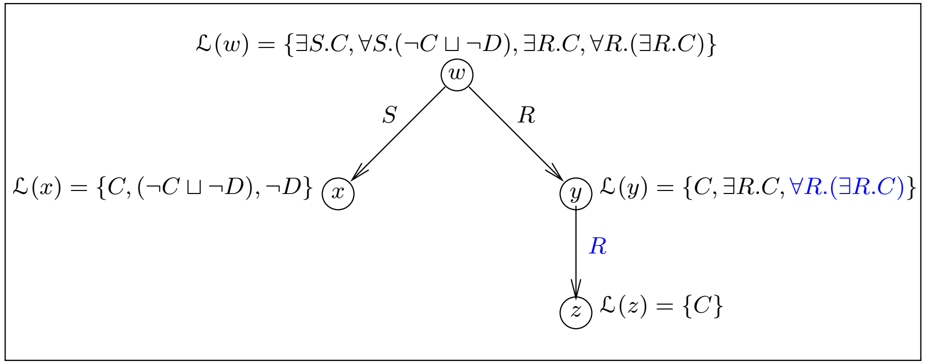

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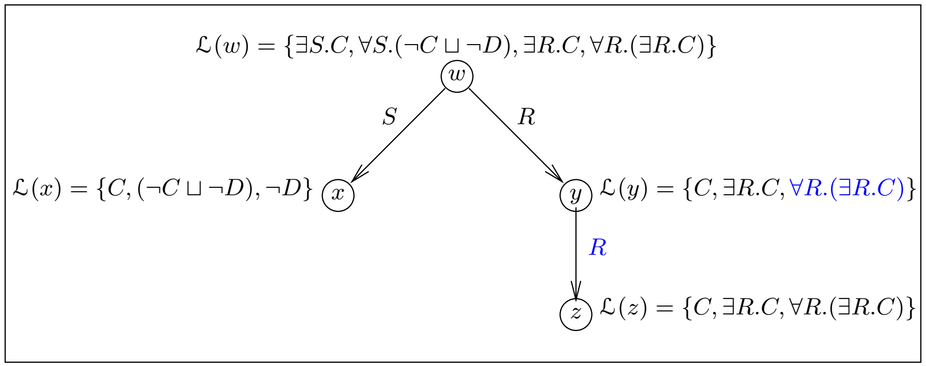

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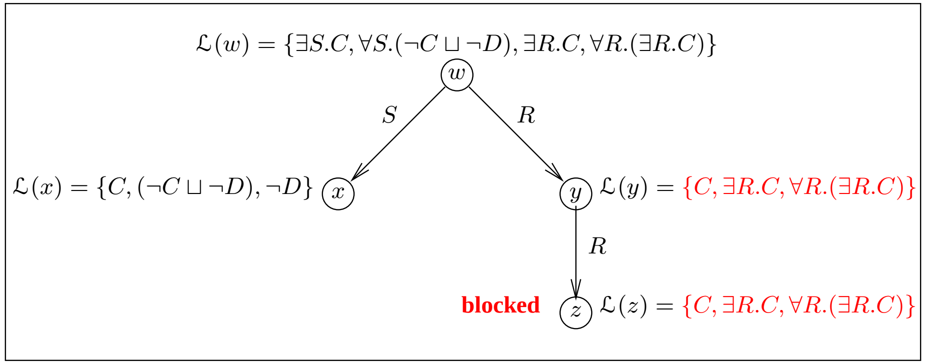

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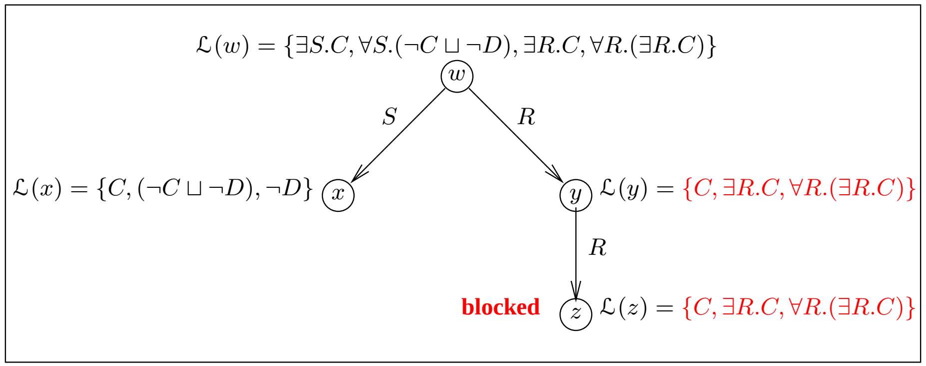
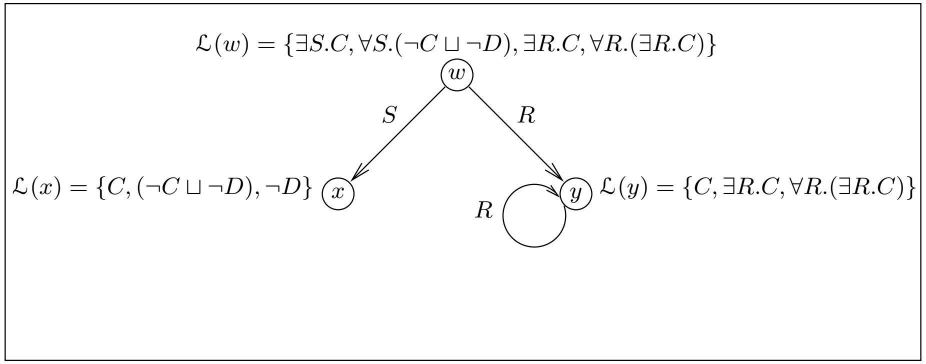


Concept is **satisfiable**: $\mathbf{T}$ corresponds to **model**

Tableaux Algorithm — Example

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role



Concept is **satisfiable**: $\mathbf{T}$ corresponds to **model**

Properties of our tableau algorithm for $\mathcal{ALC}$ with TBoxes

Lemma: Let $\mathcal{T}$ be a general $\mathcal{ALC}$-Tbox and C_0 an $\mathcal{ALC}$-concept. Then

1. the algorithm terminates when applied to $\mathcal{T}$ and C_0 and
2. the rules can be applied such that they generate a clash-free and complete completion tree iff C_0 is satisfiable w.r.t. $\mathcal{T}$.

Corollary:

1. Satisfiability of $\mathcal{ALC}$-concept w.r.t. TBoxes is decidable
2. $\mathcal{ALC}$ with TBoxes has the finite model property
3. $\mathcal{ALC}$ with TBoxes has the tree model property

A tableau algorithm for $\mathcal{ALC}$ with general TBoxes: Summary

The tableau algorithm presented here

- **decides** satisfiability of $\mathcal{ALC}$-concepts w.r.t. TBoxes, and thus also
- **decides** subsumption of $\mathcal{ALC}$-concepts w.r.t. TBoxes
- uses **blocking** to ensure termination, and
- is **non-deterministic** due to the $\rightarrow_{\sqcup}$-rule
- in the worst case, it builds a tree of depth exponential in the size of the input, and thus of double exponential size. Hence it runs in (worst case) 2NExpTime,
- can be implemented in various ways,
  - order/priorities of rules
  - data structure
  - etc.
- is amenable to optimisations – more on this next week

Summary

- **Description Logics** are family of logical KR formalisms
- **Applications** of DLs include DataBases and **Semantic Web**
  - Ontologies will provide vocabulary for semantic markup
  - OWL web ontology language based on $\mathcal{SHIQ}$ DL
  - Set to become W3C standard (OWL) & already widely adopted
  - Use of DL provides formal foundations and reasoning support
- **DL Reasoning** based on tableau algorithms
- **Highly Optimised** implementations used in DL systems
- **Challenges** remain
  - Reasoning with full OWL language
  - (Convincing) demonstration(s) of scalability
  - New reasoning tasks
  - Development of (high quality) tools and infrastructure

Challenges

- **Increased expressive power**
  - Existing DL systems implement (at most) $\mathcal{SHIQ}$
  - OWL extends $\mathcal{SHIQ}$ with datatypes and nominals
- **Scalability**
  - Very large KBs
  - Reasoning with (very large numbers of) individuals
- **Other reasoning tasks**
  - Querying
  - Matching
  - Least common subsumer
  - ...
- **Tools and Infrastructure**
  - Support for large scale ontological engineering and deployment

Resources

Slides from this talk

http://www.cs.man.ac.uk/~horrocks/Slides/Innsbruck-tutorial/

FaCT system (open source)

http://www.cs.man.ac.uk/FaCT/

OilEd (open source)

http://oiled.man.ac.uk/

OIL

http://www.ontoknowledge.org/oil/

W3C Web-Ontology (WebOnt) working group (OWL)

http://www.w3.org/2001/sw/WebOnt/

DL Handbook, Cambridge University Press

http://books.cambridge.org/0521781760.htm