## Listing closed sets of strongly accessible set systems with applications to data mining\*

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## Abstract

We study the problem of listing all closed sets of a closure operator  $\sigma$  that is a partial function on the power set of some finite ground set E, i.e.,  $\sigma : \mathcal{F} \to \mathcal{F}$  with  $\mathcal{F} \subseteq \mathcal{P}(E)$ . A very simple divide-and-conquer algorithm is analyzed that correctly solves this problem if and only if the domain of the closure operator is a strongly accessible set system. Strong accessibility is a strict relaxation of greedoids as well as of independence systems. This algorithm turns out to have delay  $O(|E|(T_{\mathcal{F}} + T_{\sigma} + |E|))$  and space  $O(|E| + S_{\mathcal{F}}S_{\sigma})$ , where  $T_{\mathcal{F}}, S_{\mathcal{F}}, T_{\sigma}$ , and  $S_{\sigma}$  are the time and space complexities of checking membership in  $\mathcal{F}$  and computing  $\sigma$ , respectively. In contrast, we show that the problem becomes intractable for accessible set systems. We relate our results to the data mining problem of listing all support-closed patterns of a dataset and show that there is a corresponding closure operator for all datasets if and only if the set system satisfies a certain confluence property. submissions.

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